# Security Design, Informed Intermediation, and the Resolution of Borrowers' Financial Distress*

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## Abstract

Banks can make efficient liquidation versus continuation decisions for their distressed loans thanks to their private information about the borrowers. Yet, their information also creates an adverse selection problem when they seek funds from uninformed investors. We show that, banks with higher-quality loans have incentives to bias their resolution polices more towards liquidation, because they optimally issue debt securities. More generally, banks can lower their costs of funding by biasing the resolution policies towards liquidation (continuation), if the issued security is restricted to be concave (convex). Regulations aimed at promoting efficient resolution of borrowers' financial distress may increase banks' funding costs, discourage their ex ante screening effort, and reduce welfare. (*JEL:* D8, G21, G23, G24)

**Keywords.** Security design, resolution of financial distress, liquidation, continuation, asymmetric information

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# 1 Introduction

The aftermath of the financial crisis in 2007-2009 has put the resolution of borrowers' financial distress back to the spotlight. For example, the amount of public company assets entering Chapter 11 bankruptcy protection during the two-year period of 2008-09 were almost 20 times more than during the previous two year, with over $3.5 trillion of corporate debt in distress or in default at one point.[1] Households were severely impacted too, with over 14 million U.S. properties with foreclosure filings from 2008 to 2014.[2] Anecdotal reports and recent empirical research, in particular on securitised mortgages, have argued that the resolution of financial distress might be inefficient.[3]

Inefficient resolution of borrowers' financial distress can arise if there are information frictions between lenders and borrowers.[4] Theories of financial intermediation have proposed that banks emerge as delegated information producers, who can achieve efficient resolution of borrowers' financial distress thanks to their ability and incentives to acquire borrower-specific, 'soft' information (e.g. Berlin and Loeys (1988), Rajan (1992), Chemmanur and Fulghieri (1994) and Bolton and Freixas (2000)).[5] Yet, banks' information about their loans creates an adverse selection problem when they need to raise funds from uninformed investors (e.g. Winton (2003) and DeMarzo (2005)). Indeed, banks routinely need funds to finance borrowers and to comply with regulatory requirements. In this paper we investigate banks' incentives to efficiently resolve borrowers' financial distress in the presence of adverse selection friction in banks' funding. Do banks resolve the financial distress of their borrowers efficiently? If not, are banks biased towards liquidation or towards continuation?

We present a model in which an informed bank determines the resolution policy for its loans in case of borrowers' financial distress, and raises funds from uninformed investors by selling a security backed by the loans.[6] We find that the bank's funding needs and private information create incentives for it to distort its resolution policy. Furthermore, the optimal distortion is

[1] Source: Gilson (2012)

[2] Source: RealtyTrac (2015).

[3] Related empirics are discussed in the empirical implications part in the introduction.

[4] The information frictions between lenders and borrowers and the associated inefficiency in the resolution of borrower financial distress can be found in Haugen and Senbet (1978), Giammarino (1989), Gertner and Scharfstein (1991), Repullo and Suarez (1998) and Wang et al. (2002).

[5] Leland and Pyle (1977) and Diamond (1984) first point out that a bank could emerge endogenously as delegated information producer to economise on investment in information.

[6] Begley and Purnanandam (2017) and Balasubramanyan et al. (2017) provide evidence that banks use their private information about loan quality when designing the securities in mortgage securitisation and in syndicated loan market respectively.

towards liquidation, as opposed to continuation, because the bank raises funds in the form of debt, which is indeed the optimal security. Our results provide novel reasons for why policies aimed at correcting inefficiency in resolution, e.g. excessive foreclosures in mortgages, could be socially costly, by increasing banks' funding cost and weakening their screening incentives.

**Model preview.** The bank has a continuum of ex ante identical assets (a "pool" of loans to borrowers). Upon realisation of an aggregate shock (the bad state), some borrowers enter into financial distress – "default". The bank's resolution policy determines whether a distressed loan is liquidated or continued. Liquidation delivers a safe cash flow from the immediate sale of the loan or the underlying collateral, whereas continuation may result in a higher (lower) cash flow if the borrower recovers (re-defaults), depending on the realisation of a future aggregate shock. The efficient resolution policy is to liquidate all distressed loans that have a higher liquidation value than the expected recovery value, and continue the rest.

To meet its funding needs, the bank designs and sells a security backed by the cash flow from the loan pool to competitive investors (as in DeMarzo (2005)), and chooses the resolution policy for the loans. A key friction in our model is that investors are less informed than the bank about the quality of the loans, particularly regarding the default probability of the borrowers.

**Results preview.** In the benchmark case with symmetric information, the bank chooses the efficient resolution policy and reaps the full value of the loans immediately by simply selling a pass-through security to investors. In other words, if the market for funds is frictionless, financing is irrelevant for the bank's resolution policy.

Our main result is that the adverse selection problem at the bank's financing stage could cause the bank with higher-quality, less distress-prone loans (the high type) to optimally bias its resolution policy for its borrowers towards liquidation, because it endogenously designs and sells a debt security to uninformed investors. In line with the literature on security design, the bank retains the residual levered equity claim to signal its quality.[7,8]

The optimality of a liquidation bias in resolution policy comes from two channels, both

[7] DeMarzo et al. (2015) shows that in an ex-post liquidity-based security design game like ours, standard debt is the least information sensitive and thus the optimal monotone security when the cash flow satisfies Hazard Rate Ordering (HRO) property. See also Chemla and Hennessy (2014) and Vanasco (2016) for recent theoretical works with costly retention of the equity tranche as signals. Empirically, Begley and Purnanandam (2017) find that conditional on observable characteristics, RMBS deals with larger equity tranche have lower delinquency rate and command higher prices, suggesting that the signalling mechanism is at play.

[8] As also discussed in Begley and Purnanandam (2017), even if the bank sells off the equity tranche at a later date, the initial retention of the equity tranche could still be a costly signal because i) the opportunity cost of the locked-up capital could still be significant in a high-growth market and ii) the equity tranches in practice are often sold to sophisticated and informed investors like hedge funds and mutual fund managers, who are likely to have stronger bargaining power and/or scarcer capital than uninformed senior tranche investors.

stemming from that fact that more liquidation reduces the riskiness of the cash flow from the loan pool by limiting the loan pool's exposure to borrowers' re-default risks. First, even under symmetric information, safer cash flow increases the expected value of a given debt (concave) security, by Jensen's inequality, allowing the bank to realise greater gains from trade with the investors. Second, under asymmetric information, further biases towards liquidation could serve as a second costly signal. This is because liquidation satisfies the single-crossing condition: safer cash flow reduces the value of the bank's retained equity (convex) claim and crucially, more so for banks with lower-quality, more-distress-prone, loans.

The above intuition holds for any exogenously given security design. A distortion in the bank's resolution policy towards liquidation (continuation) is optimal if the security issued is concave (convex) in the cash flow in the bad state. Our analysis thus reveals that, the *direction* of the distortion in the resolution policy depends on the security issued. We believe these insights are general and can be applied to other economic settings.

Overall, our analysis suggests that the adverse selection frictions in banks' financing causes it to issue debt and hence to distort its resolution policy towards liquidation. A comparative-statics analysis confirms that when information asymmetry is more severe, the liquidation bias in the high-type bank is stronger. An extension to a multiple-type model reveals that while the liquidation bias is increasing in loans' quality, interestingly, the size of the retained equity claim need not be. The latter result differs from and complements the canonical security design models, as we show that the distortion in resolution policy could partially substitute retention as a signal of quality.

**Extension and Policy.** We extend the model to allow the bank to ex ante exert screening effort at loan origination to increase the likelihood of creating a high-quality loan pool. We find that adverse selection in the bank's funding discourages the bank from screening diligently because part of the gains from having a high-quality pool is lost to the costly retention. Nonetheless, a resolution policy biased towards liquidation mitigates such inefficiency and in turn restores some of the weakened screening incentives.

The main policy implication from our results is a cautionary one in the Lucas' critique fashion: policies aimed at promoting the efficient resolution of distressed loans, such as the Home Affordable Modification Program (HAMP) for mortgages, could inadvertently *reduce* the bank's incentive to screen loans diligently, leading to lower average loan quality and overall welfare.[9] The analysis suggests that, while policies mitigating liquidations might be warranted due to some negative externalities not considered in our model, policy makers should take into account their potential effects on loan origination and on banks' funding costs.

**Empirical implications.** Our model generates several novel empirical predictions. Our main result predicts an average liquidation bias in banks' resolution policy, consistent with Maturana (2017).[10] To the extent that banks rely on mortgage servicers to carry out the resolution of borrowers' financial distress, our model is consistent with the evidence that mortgage servicers have biased incentives towards liquidation (Thompson (2009), Kruger (2016)). Moreover, our model predicts that, ceteris paribus, the liquidation bias in banks' resolution policy and the associated loan losses should be larger for banks with higher-quality loans.[11] Finally, because banks signal quality through a combination of security design and liquidation bias in the resolution policy, the size of retention may be non-monotonic in the quality of the loans. This can reconcile the empirical literature that tests retention as a signal of quality and finds mixed results.[12]

**Contribution to the literature.** To our best knowledge, this paper is the first to study the role played by banks' resolution policy of their borrowers' financial distress in mitigating the adverse selection friction in banks' financing. Our paper therefore complements the analysis of one of the fundamental roles of banks as delegated information producers to achieve efficient monitoring (Diamond (1984)) and resolution of borrowers' financial distress (Berlin and Loeys (1988), Chemmanur and Fulghieri (1994) and Bolton and Freixas (2000)), by acknowledging their limits due to adverse selection frictions in banks' financing.[13]

Some important elements of our paper can be found in Winton (2003). Winton also observes that a bank's private information regarding its loans enables efficient resolution but also increase

[9] In the aftermath of the subprime mortgage crisis, the U.S. experienced a large number of delinquencies and foreclosure filings. In response, the U.S. government developed the HAMP to incentivise mortgage modification instead of foreclosure, by providing direct one-off and annual monetary incentives to mortgage servicers for each successfully modified delinquent mortgage. For a detailed description and an empirical evaluation of HAMP, see Agarwal et al. (2012a).

[10] Maturana (2017) shows that in the context of securitised mortgages, the marginal returns of liquidation exceeds that of restructuring, suggesting that the associated loan loss could have been smaller if there were fewer liquidations.

[11] Testing signalling models is challenging due to the unobservable nature of private information. Recent studies like Begley and Purnanandam (2017) has proxied the ex ante unobservable pool quality with ex post *abnormal* default rate. Our cross-sectional prediction about the resolution policy can be tested using similar methodology.

[12] Garmaise and Moskowitz (2004) and Agarwal et al. (2012b) fail to find strong evidence of retention as a signal of quality, while Begley and Purnanandam (2017) find consistent evidence in the residential mortgage-backed securities market.

[13] Rajan (1992) highlights another dark side of having an informed lender that the lender cannot commit not to extract rent from the borrower with its private information by threatening to liquidate the firm.

its funding costs due to adverse selection. The focus of Winton, however, is different from ours. We study the bank's optimal funding structure and show that the bank adopts a distorted resolution policy in order to reduce its funding cost, whereas Winton assumes that the bank issues equity and instead emphasises that the bank should hold the borrowers' debt to reduce its funding cost.

Our paper extends the canonical liquidity-based security design models, such as DeMarzo (2005) and Biais and Mariotti (2005), by allowing the informed issuer to take actions that affect the distribution of the underlying asset's cash flow, specifically by choosing the resolution policy of borrowers' financial distress. We contribute to this literature by showing that distortions in the bank's resolution policy could substitute retention as a signal of quality.

In terms of application, our paper contributes to the burgeoning literature on the interaction between the financing problem of banks and banks' roles as informed intermediaries. Chemla and Hennessy (2014) and Vanasco (2016) explore the trade-off between secondary market liquidity and the bank's incentives to originate good assets. In addition to inefficient screening, we show that banks' resolution policy is also distorted, in order to alleviate the inefficiencies caused by adverse selection.

**Roadmap.** The rest of the paper is organised as follows. Section 2 describes the model setup. Section 3 carries out the main analysis of the equilibrium with endogenous security design and resolution policy. Section 4 extends the model to consider ex ante screening incentives of the bank. Section 5 studies an extended model with multiple types. Section 6 concludes.[14]

## 2 Model setup

There are three dates in the baseline model: $t = 1$, 2 and 3.[15] The model's participants consist of a bank who owns a continuum of loans and competitive outside investors. All agents are risk neutral. The bank has a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. Outside investors are deep-pocketed and have a discount factor equal to 1. Hence, there are gains from trade between the bank and the investors. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the bank's funding needs.[16]

[14]The Internet Appendix associated with the paper can be found at https://goo.gl/AfUq35

[15]We extend the model to a loan-origination stage $t = 0$ in Section 4.

[16]Modelling gains from trade as a discount factor $\delta < 1$ is standard in the literature to capture liquidity needs stemming from, e.g., capital constraints, new investment opportunities, risk-sharing, etc. (see Holmström and Tirole (2011)).

### Loan pool and resolution policy

The bank owns a loan pool containing a continuum of ex ante identical loans that pay off at $t = 3$. We model the loan pool as a well-diversified portfolio of loans. The loan pool is thus only exposed to aggregate risks, which affect the ability for all borrowers to repay.[17] Specifically, with probability $\pi$, the loan pool is in the good state ($G$) at $t = 2$ and no borrowers default. In the good state, each loan returns a riskless cash flow $Z > 0$ at $t = 3$. With probability $1 - \pi$, the loan pool is in the bad state ($B$) at $t = 2$ and each borrower defaults with some i.i.d. probability $d$. Thanks to the diversification benefit, the proportion of the loans that become distressed at $t = 2$ is also $d$. The remaining performing loans continue to return a riskless cash flow $(1 - d)Z$ at $t = 3$.

When a fraction $d$ of the loans become distressed in the bad state at $t = 2$, the bank can choose to liquidate a fraction $\lambda$ of the distressed loans and continue the remaining fraction $1 - \lambda$. We will henceforth refer to $\lambda \in [0, 1]$ as the bank's resolution policy. If a distressed loan is liquidated, the loan is terminated and the collateral asset is sold to outside investors. Let $\mathcal{L}(\lambda)$ denote the total liquidation proceeds. Alternatively, continuation gives a cash flow $X > 0$ with probability $\theta$ at $t = 3$ (recovery) or zero otherwise (re-default). For simplicity, we assume that the recovery (and re-default) of continued loans in a given pool are perfectly correlated. This is also in line with the assumption of a well-diversified loan pool so that only aggregate risks affect the repayment of the borrowers. Finally, we assume that $X \leq Z$, so that the payoff of a loan in the good state is at least as high as in a bad state, even if the loan resumes payment. Intuitively, the difference may account for the value of the temporary missing payments and the reduced repayments after renegotiations.

The exact functional form of the total liquidation proceeds $\mathcal{L}(\lambda)$ depends on characteristics of the loans as well as the direct and indirect costs associated with liquidation. We abstract from these considerations to keep the analysis general and make the following assumption on the liquidation technology.

**Assumption 1.** *For* $\lambda \in [0, 1]$, *(i)* $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} > 0 > \frac{\partial^2 \mathcal{L}(\lambda)}{\partial \lambda^2}$ *and (ii)* $\lim_{\lambda \to 0^-} \frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} > \theta X > \lim_{\lambda \to 1^+} \frac{\partial \mathcal{L}(\lambda)}{\partial \lambda}$.

Part *(i)* of Assumption 1 states that the total liquidation proceeds increases at a decreasing rate in the fraction of loans liquidated. Part *(ii)* of Assumption 1 implies that the efficient

[17] Such aggregate risks can be aggregate property prices or employment opportunities for the borrowers.

resolution policy that maximises the expected value of distressed loans involves some liquidations and some continuations. Intuitively, Assumption 1 is satisfied if the distressed loans have heterogeneous liquidation values (with those with highest liquidation values being liquidated first), which in turn could be motivated by heterogeneity in collateral values of the distressed loans or in the costs associated with liquidation.[18] Heterogeneity in continuation payoffs could be introduced but does not bring any additional insights.

The loan pool's exposure to aggregate risks is characterised by the probability of entering state $G$. This probability $\pi \in \{\pi_H, \pi_L\}$, where $\pi_H > \pi_L$, is loan-pool specific and is the source of information asymmetry between the bank and outside investors, as detailed in the next section. We interpret $\pi_i$ as the "quality" of the loan pool and thus the "type" of the bank (subscript "H" stands for "High" and "L" for "Low").[19] The assumption that the delinquency rate of a loan pool being the bank's private information is in line with empirical studies such as Begley and Purnanandam (2017).[20] A high-quality pool is less exposed or more resilient to aggregate risks and hence is more likely to have no distressed loan (to be in the good state $G$). At $t = 1$, all model participants have the prior belief that $\pi = \pi_H$ with probability $\gamma$.[21] One interpretation of $\gamma$ is a publicly observable signal about the quality of the loans in the pool, e.g. the average FICO scores of the borrowers in a loan pool. Therefore, a pool with higher $\gamma$ is observably better because it is more likely to be a high-quality pool.

To summarise, for a given resolution policy $\lambda$, the overall cash flow $c$ from a type $i$ loan pool at $t = 3$ is given by $c_1 \equiv Z$ with probability $\pi_i$ (in the "Good" state), $c_2(\lambda) \equiv (1-d)Z + d[\mathcal{L}(\lambda) + (1-\lambda)X]$ with probability $(1-\pi_i)\theta$ (in the "Recovery" state), and $c_3(\lambda) \equiv (1-d)Z + d\,\mathcal{L}(\lambda)$ with probability $(1-\pi_i)(1-\theta)$ (in the "Re-default" state), as illustrated in Figure 1.

### Financing and security design

Because of the liquidity discount $\delta$, at $t = 1$, the bank would like to raise cash by selling a security backed by the cash flow of the loan pool to outside investors. The bank receives the

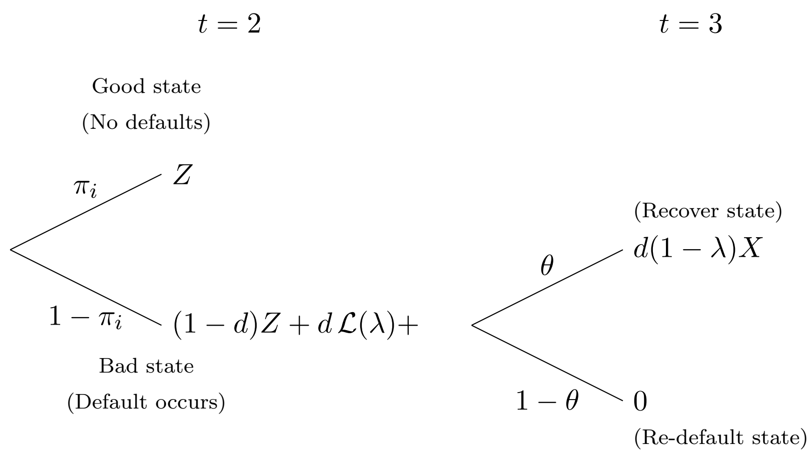
[18]A microfoundation of such a liquidation technology based on heterogeneous liquidation values is provided in the Internet Appendix, where we show that it is indeed optimal for the bank to liquidate the distressed loans with higher liquidation values first.

[19]We extend the baseline two-type model to a multiple-type one in Section 5.

[20]In addition, we could also allow the bank to have some private information on the liquidation proceeds $\mathcal{L}$ and the recovery probability $\theta$. It complicates the analysis without much additional insights as in that case the efficient resolution benchmark will depend on types as well. For simplicity, we assume information about $\mathcal{L}$ and $\theta$ is symmetric.

[21]In Section 4 we endogenise this probability $\gamma$ in the loan-origination stage $t = 0$ through the bank's screening effort choice.

Figure 1: Loan pool cash flow for a given resolution policy $\lambda$




cash proceeds from selling the security at $t = 1$, and retains any residual cash flow from the loan pool after paying off the investors at $t = 3$.

We mentioned earlier that there is asymmetric information between the bank and the investors. This creates a financing friction for the bank akin to the classical lemon's problem in Akerlof (1970). Specifically, at the beginning of $t = 1$, the bank receives private information regarding the quality of the mortage pool $\pi_i \in \{\pi_H, \pi_L\}$. This for example reflects the bank's soft information about its borrowers.

We model the financing stage as follows, similar to the ex-post security design problem in DeMarzo (2005).[22] After observing the private information $\pi_i$, the bank offers outside investors a security $\mathcal{F}$ and promises a resolution policy $\lambda$. The security $\mathcal{F}$ is contracted upon the cash flow of the loan pool at $t = 3$, specifying a payment $f(c)$ to investors for each realisation of the cash flow $c \in \{c_1, c_2(\lambda), c_3(\lambda)\}$. The security can be expressed as $\mathcal{F} = \{f_1, f_2(c_2), f_3(c_2)\}$, specifying the payments to investors given the cash flow realisation in the good, recovery and re-default state respectively. Table 1 summarises the possible realisations of the cash flow and the corresponding payments specified by a security. As common in the security design literature, we restrict our attention to monotone securities subject to limited liability.[23] The value of the

[22] DeMarzo and Duffie (1999) solves the ex ante security design problem, whereas we solve for the ex post security design problem after the banks learn about their private information. As shown by DeMarzo (2005) and DeMarzo et al. (2015), similar intuition carries through in the ex post problem, although the problem becomes more complicated as the design itself becomes a signal.

[23] A monotone security satisfies that, a higher realisation of the loan pool cash flow should leave both the outside investors and the bank a (weakly) higher payoff. Although this implies some loss of generality, it is not uncommon in the security design literature, e.g. Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the loan pool if the security leaves the issuer a payoff that is not increasing in the cash flow.

Table 1: Payoffs of a generic security backed by the loan pool cash flow

| States | Realisation of cash flow | Security payoff $\mathcal{F}$ |
|---|---|---|
| Good (no defaults) | $c_1 \equiv Z$ | $f_1$ |
| Recovery | $c_2(\lambda) \equiv (1-d)Z + d\,\mathcal{L}(\lambda) + d(1-\lambda)X$ | $f_2(c_2(\lambda))$ |
| Re-defaults | $c_3(\lambda) \equiv (1-d)Z + d\,\mathcal{L}(\lambda)$ | $f_3(c_3(\lambda))$ |

security $\mathcal{F}$ backed by a loan pool of quality $\pi_i$, given a resolution policy $\lambda$, is thus given by

$$p_i(\mathcal{F}, \lambda) = \pi_i f_1 + (1-\pi_i)[\theta f_2(c_2(\lambda)) + (1-\theta) f_3(c_3(\lambda))] \tag{1}$$

And the value of the loan pool $V_i$ is

$$V_i(\lambda) = \pi_i c_1 + (1-\pi_i)[\theta c_2(\lambda) + (1-\theta) c_3(\lambda)] \tag{2}$$

After observing the offer $(\mathcal{F}, \lambda)$, the competitive investors form a posterior belief $\hat{\pi}$ regarding the quality of the loan pool, and bid the price of the security $p$ up to its fair value given the belief, $p_{\hat{\pi}}(\mathcal{F}, \lambda)$, defined analogously to Eq. 1. At $t = 3$, after paying investors according to $\mathcal{F}$ from the loan pool cash flow, the bank consumes any residual cash flow. Therefore, the expected payoff of a bank with loan quality $\pi_i$ is

$$U_i(\mathcal{F}, \lambda; \hat{\pi}) = p_{\hat{\pi}}(\mathcal{F}, \lambda) + \delta\,[V_i(\lambda) - p_i(\mathcal{F}, \lambda)] \tag{3}$$

which is the sum of the proceeds from selling the security to investors and the discounted value of retained cash flow.

### Timeline and the equilibrium concept

The timeline of the model is summarised in Table 2. The main analysis of the baseline model concerns only $t = 1$, 2 and 3. We extend the model to consider loan origination at $t = 0$ in Section 4.

The equilibrium concept in this model is the perfect Bayesian equilibrium (PBE). Formally, a PBE consists of a security $\mathcal{F}_i$ issued by the bank of each type $i \in \{H, L\}$, the resolution policy $\lambda_i$ of the bank of each type, and a system of beliefs such that i) the bank chooses the security and the resolution policy at $t = 1$ to maximise its expected payoff, given the equilibrium

The full characterisation of a monotone security subject to limited liability is given in Appendix A.

Table 2: Model timeline

| | |
|---|---|
| $t = 0$ | Bank exerts screening effort (Section 4 only) |
| $t = 1$ | Bank observes $\pi_i$ and offers $(\mathcal{F}_i, \lambda_i)$<br>Investors purchase the security $\mathcal{F}_i$ at price $p$ |
| $t = 2$ | Loan defaults occur in Bad state<br>Distressed loans are liquidated or restructured according to the resolution policy $\lambda_i$ |
| $t = 3$ | Loan pool cash flow is realised<br>Bank and investors are paid off |

choices of the other agents and the equilibrium beliefs, and ii) the beliefs are rational given the equilibrium choices of the agents and are formed using Bayes' rule (whenever applicable). As there can be multiple equilibria in games of asymmetric information, we invoke the Intuitive Criterion of Cho and Kreps (1987) to eliminate equilibria with unreasonable out-of-equilibrium beliefs. This allows us to eliminate all but the least cost separating equilibrium (as shown in Lemma 2).

### Discussion of the framework

We adopt and extend the liquidity-based security design framework developed by DeMarzo (2005) to study the joint optimisation problem of security design and the resolution of borrowers' financial distress under asymmetric information. As we shall later illustrate, our mechanism involves the retention of the residual equity claim as a signal of quality, the observability of the bank's resolution policy, and the bank's ability to commit to an ex post inefficient resolution policy. We believe our framework captures some realistic aspects of the securitisation market and here we will discuss some of the crucial features.

*On the evidence on equity tranche as a signal*

As in DeMarzo (2005), the optimal security in our model is risky debt. In other words, banks with high-quality loan pools signal information to investors through the retention of the residual equity claim. The signalling mechanism is supported by empirical evidence in the residential mortgage market.[24] For example, Begley and Purnanandam (2017) find that, conditional on observable characteristics, residential mortgage-backed securities (RMBS) deals with larger equity tranches have lower delinquency rates and command higher prices, suggesting

[24] See the discussion in footnote 7 for recent theoretical works featuring the same mechanism and for supporting evidence in the RMBS market.

that investors could and do learn from the equity tranche size.[25]

*On the observability of resolution policy and the bank's ability to commit*

In the model, the bank can commit to a resolution policy, which is observed by investors as part of the signal. We argue that this is realistic. In the mortgage context for instance, banks often delegate the resolution decisions of delinquent mortgages to third-party servicers. Banks can effectively commit to a resolution policy through either the fine-tuning of incentives in the servicers' compensation contract or the choice of servicers with different liquidation capacity. Empirical studies have shown that incentives and identity of servicers matter. Thompson (2009) and Kruger (2016) have argued and documented that the compensation of servicers overall is biased towards liquidation (foreclosure), whereas Agarwal et al. (2011), among others, have shown that the identity of servicers has explanatary power for the liquidation probability of delinquent mortgages. In practice, Moody's, a rating agency, produces "Servicer Quality" (SQ) rating which assesses RMBS and ABS servicers' loss mitigation ability in case of delinquency in securitisation (Moody's (2016)). Therefore, by observing the servicers' identity and compensation structure listed in the prospectus of the deals, investors can infer the resolution policy indirectly chosen by the issuing banks. In the Internet Appendix, we discuss these mechanisms in details and formalise them as extensions of the model.

## 3 Security design and resolution policy

In this section we carry out the main analysis of our model of security design with endogenous resolution. We start with defining the efficient resolution policy and show that in the absence of information friction, there is no distortion in resolution policy. Then we characterise the equilibrium under asymmetric information and study its implications for the endogenous security design and resolution policy.

[25] While it is possible that in practice the retained tranche might be subsequently sold off in the secondary market, the *initial* retention of the equity tranche could still signal information as long as there are substantial (opportunity) costs associated with it. First, the delayed sale of the equity tranche could be costly to the banks because it implies that some capital is locked-up and thus the banks have to forgo some profitable lending in the loan market or investments in general. Second, due to the segmentation of markets for different ABS tranches, the banks are also likely to be in a less advantageous position in the sale of equity tranche than those of the senior tranche. In practice, the typical equity tranche investors are sophisticated investors like hedge fund with whom the banks are likely to have a weaker bargaining position. For a detailed discussion, see Begley and Purnanandam (2017).

### 3.1 Symmetric-information equilibrium and efficient resolution benchmark

It is useful to start with defining the efficient resolution policy $\lambda^{EF}$, which is one that maximises the expected loan value $V_i$.

$$\lambda_i^{EF} = \arg\max_{\lambda} \; V_i(\lambda) \tag{4}$$

$\lambda^{EF}$ is characterised by the first order condition, $\frac{\partial \mathcal{L}(\lambda_i^{EF})}{\partial \lambda} = \theta X$. Intuitively, a resolution policy is efficient when the marginal proceeds from liquidation equal to the expected recovery value given continuation. Note that the efficient resolution policy is the same across loan quality and therefore we drop the subscript $i$ and simply denote by $\lambda^{EF} \in (0,1)$.[26] $\lambda^{EF}$ serves as the benchmark of our analysis, to which we later compare the equilibrium resolution policies under symmetric and asymmetric information.

In the case of symmetric information, the investors directly observe the bank's loan quality $\pi_i$. The bank designs a security $\mathcal{F}$ and adopts a resolution policy $\lambda$ to maximise its expected payoff. Its problem can be written as follows

$$U_i^{SI} \equiv \max_{\{\mathcal{F},\lambda\}} U_i(\mathcal{F},\lambda;\pi_i) = p_i(\mathcal{F},\lambda) + \delta\left[V_i(\lambda) - p_i(\mathcal{F},\lambda)\right] \tag{5}$$

We tackle the joint optimisation problem in sequence. For a given security $\mathcal{F}$, the optimal resolution policy under symmetric information $\lambda_i^{SI}(\mathcal{F})$ is defined as

$$\lambda_i^{SI}(\mathcal{F}) \equiv \arg\max_{\lambda} U_i(\mathcal{F},\lambda;\pi_i) = \arg\max_{\lambda} \delta V_i(\lambda) + (1-\delta)p_i(\mathcal{F},\lambda) \tag{6}$$

By comparing the definitions in 4 and 6, we can see that even under symmetric information, the optimal resolution policy can be different from the efficient one thanks to the bank's liquidity needs as well as the security design. Indeed, when $\delta = 1$, the bank has no liquidity needs and thus no reasons to trade with investors. The bank thus simply maximises the expected value of the loan pool, i.e. $\lambda_i^{SI}(\mathcal{F}) = \lambda^{EF}$. When $\delta < 1$, the bank values current cash flows more and it can realise some gains from trade via selling the security to investors. Hence, in order to further increase the proceeds $p_i(\mathcal{F},\lambda)$, the bank is willing to bias the resolution policy. The key observation here is that depending on the *shape* of the security $\mathcal{F}$, the optimal bias can go either way. We formalise this result in the following lemma. All proofs are in the Appendix.

[26]That $\lambda^{EF}$ is strictly interior follows from Part (*ii*) of Assumption 1.

**Lemma 1.** *For a given security $\mathcal{F}$, the optimal resolution policy under symmetric information*

- *has a liquidation bias, $\lambda^{SI}(\mathcal{F}) \geq \lambda^{EF}$, if in the bad state $\mathcal{F}$ is concave*
- *has a continuation bias, $\lambda^{SI}(\mathcal{F}) \leq \lambda^{EF}$, if in the bad state $\mathcal{F}$ is convex*
- *is efficient, $\lambda^{SI}(\mathcal{F}) = \lambda^{EF}$, if in the bad state $\mathcal{F}$ is linear*

*Note the optimal resolution policy depends on the security $\mathcal{F}$ but not on bank type $i$. If the second order condition holds, the magnitude of the bias $|\lambda^{SI}(\mathcal{F}) - \lambda^{EF}|$ is decreasing in $\delta$.*[27]

Lemma 1 comes from the different effects of liquidation and continuation on the loan pool's cash flow. Liquidation limits the the loan pool's exposure to borrowers' re-default risks, making the underlying cash flow less volatile. Thus, by Jensen's inequality, more liquidation at the margin increases the expected value of a concave security. In this case, $\lambda^{SI}(\mathcal{F})$ optimally biases towards liquidation and trades off the loss of the value of the loan pool against the gains in liquidity. The bias is therefore stronger when the bank's liquidity needs is higher (small $\delta$). By the same logic, a continuation bias is optimal when the security is convex.

Next, for a given resolution policy $\lambda$, the optimal security design $\mathcal{F}^{SI}(\lambda)$ is

$$\begin{aligned}\mathcal{F}^{SI}(\lambda) \equiv \arg\max_{\mathcal{F}} U_i(\mathcal{F}, \lambda; \pi_i) &= \arg\max_{\mathcal{F}} p_i(\mathcal{F}, \lambda) \\ &= \arg\max_{\mathcal{F}} \pi_i f_1 + (1-\pi_i)[\theta f_2(c_2(\lambda)) + (1-\theta) f_3(c_3(\lambda))] \qquad (7)\end{aligned}$$

In words, for any given resolution policy, the bank chooses the security that maximises the proceeds from sale. The optimal security is thus a pass-through security which promises all the underlying cash flows to investors, i.e. $\mathcal{F}^{SI}(\lambda) = \{c_1, c_2(\lambda), c_3(\lambda)\}$. Intuitively, under symmetric information, any security would be sold at its expected value. A bank with liquidity needs should simply sell all the cash flow from the loan pool. Note that the pass-through security is linear in the underlying loan pool's cash flow.

We are now ready to characterise fully the symmetric-information equilibrium in the following proposition.

**Proposition 1.** *In the symmetric-information equilibrium, the bank of both types sells the entire cash flow of the loan pool to investors, or equivalently issues a pass-through security and chooses the efficient resolution policy.*

[27] We provide a sufficient condition for the bias to be strict in the proof of Lemma 1.

Proposition 1 is immediate. Given the optimal security is a pass-through security, which is linear in the loan pool's cash flow, by Lemma 1, the optimal resolution policy under symmetric information is the efficient one. Therefore, any distortion in the equilibrium resolution policy in this paper will be driven by information asymmetry between the bank and the investors.

### 3.2 Asymmetric-information equilibrium

The bank's private information about the quality of the loan pool creates an adverse selection problem when the bank raises financing from uninformed investors. In this section, we show how the bank optimises its security design and resolution policy to mitigate the adverse selection problem. The analysis leads to the main result of the paper: biasing the resolution policy towards liquidation is optimal because the optimal security is debt.

Our analysis focuses on the least cost separating equilibrium, which is the unique equilibrium that satisfies the Intuitive Criterion in our model. We state this uniqueness result in the following lemma and formally prove it in the Appendix.

**Lemma 2.** *The unique equilibrium that satisfies the Intuitive Criterion is the least cost separating equilibrium.*

Let's start the analysis with the bank who owns a low-quality, i.e. a more distress-prone loan pool. In the least cost separating equilibrium, the low-type bank achieves the symmetric-information outcome because the high-type bank has no incentive to mimic the low type (verified in equilibrium). Denote by $U_i^*$, $\lambda_i^*$ and $\mathcal{F}_i^*$ the expected payoff, the resolution policy, and security design of a type $i$ bank in equilibrium respectively. Therefore, $U_L^* = V_L(\lambda^{EF})$, $\lambda_L^* = \lambda^{EF}$, and $\mathcal{F}_L^* = (c_1, c_2(\lambda^{EF}), c_3(\lambda^{EF}))$. There is no distortion in the low-type bank's resolution policy.

Meanwhile, in order to deter the low type's mimicry, the high-type bank has to take costly actions such as retention of the loan pool's cash flow and distortion in the resolution policy. More precisely, in the least cost separating equilibrium, the high-type bank's expected payoff is maximised by the choices of the monotone security $\mathcal{F}_H$ under limited liability and the resolution policy $\lambda_H$, subject to the incentive compatibility constraint ($IC$) that the low type does not mimic. The maximisation problem is stated below.[28]

$$
\begin{aligned}
&U_H^* = max_{\{\mathcal{F}_H,\lambda_H\}}\, U_H(\mathcal{F}_H,\lambda_H;\pi_H) = p_H(\mathcal{F}_H,\lambda_H) + \delta[V_H(\lambda_H) - p_H(\mathcal{F}_H,\lambda_H)] \\
&s.t. \quad (IC) \quad U_L^* \geq U_L(\mathcal{F}_H,\lambda_H;\pi_H) = p_H(\mathcal{F}_H,\lambda_H) + \delta[V_L(\lambda_H) - p_L(\mathcal{F}_H,\lambda_H)]
\end{aligned} \tag{8}
$$

The (IC) ensures that the low-type bank's equilibrium payoff is higher than its mimicking payoff. $U_L(\mathcal{F}_H,\lambda_H;\pi_H)$ represents the low type's payoff when it deviates to the high type's policies $\{\mathcal{F}_H,\lambda_H\}$. When it does so, the investors believe that it has a high quality loan pool ($\hat{\pi} = \pi_H$) and buy the security that is worth $p_L(\mathcal{F}_H,\lambda_H)$ at the higher price $p_H(\mathcal{F}_H,\lambda_H)$.

Besides the liquidity motives studied in the case of symmetric information, the high-type bank's choices of the security and the resolution policy here are also driven by signalling concerns. Like before, we tackle the joint optimisation problem in two steps. We start with the effect of the resolution policy $\lambda$ for a given security $\mathcal{F}$. In the following lemma, we establish the signalling role of distortiong in resolution policy. Furthermore, and crucially, we find that the security design $\mathcal{F}$ determines when biases towards liquidation or continuation (or both) could serve as signals of quality. A distortion in the resolution policy could serve as a costly signal if the single crossing condition holds: the distortion is costly for all banks and costlier for the mimicking lower-quality banks.

**Lemma 3.** *For any security design $\mathcal{F} \neq \{f_1, c_2(\lambda), c_3(\lambda)\}$, distorting the resolution policy beyond the symmetric-information benchmark*

- *towards liquidation ($\lambda \geq \lambda^{SI}(\mathcal{F})$) could signal quality if in the bad state $\mathcal{F}$ is concave*
- *towards continuation ($\lambda \leq \lambda^{SI}(\mathcal{F})$) could signal quality if in the bad state $\mathcal{F}$ is convex*
- *towards both liquidation and continuation could signal quality if in the bad state $\mathcal{F}$ is linear.*

*For $\mathcal{F} = \{f_1, c_2(\lambda), c_3(\lambda)\}$, distortion in neither direction could signal quality.*

Intuitively, when the security sold to investors is concave, further distorting the resolution towards liquidation is 1) costly, due to Jensen's inequality, because the retained claim is convex and 2) costlier to the mimicking banks with lower-quality loan pools because they default more often. Similarly, when the issued security is convex, more continuation hurts all types of banks

[28] The characterisation of the monotonicity and limited liability constraints on the security design is in Appendix A.

but the mimicking lower-quality banks more. Finally, if the bank retains no cash flow in the bad state, distortion in the resolution policy is not costly and thus could not signal quality.

Next, we characterise the optimal security for any given resolution policy.

**Lemma 4.** *For any resolution policy $\lambda_H$, risky debt with a promised repayment $F_H(\lambda_H) \in (c_3(\lambda_H), Z)$ implements the optimal security for the high-type bank.*

This result is consistent with the classic literature on the pecking order of external financing under asymmetric information (e.g. Myers (1984)).[29] The optimal monotone security issued by the high type is a debt security, because it is least information sensitive. The high type exhausts its capacity to issue risk-free debt ($F_H(\lambda) > c_3(\lambda)$), which is free from any information asymmetry. The retention of future cash flow being a costly signal is a well-established result in the security design literature such as Leland and Pyle (1977) and DeMarzo and Duffie (1999).

We can now proceed to characterise the joint determination of the optimal resolution policy and the optimal security stated in Eq. 8. Let $(\mathcal{F}_i^*, \lambda_i^*)$ denote the equilibrium security and resolution of a type $i$ bank. We have the following proposition, which is the main result of our paper.

**Proposition 2.** *In the least cost separating equilibrium, the low-type bank sells the entire cash flow and adopts the efficient resolution policy $\lambda_L^* = \lambda^{EF}$, whereas the high-type bank issues a risky debt with a promised repayment $F_H^* = F_H(\lambda_H^*) \in (c_3(\lambda_H^*), Z)$ and adopts a resolution policy with a liquidation bias $\lambda_H^* \geq \lambda^{SI}(\mathcal{F}_H^*) \geq \lambda^{EF}$. The second inequality is strict if and only if $G(\lambda^{EF}) > 0$, equivalently $F_H^* < c_2(\lambda^{EF})$, where*

$$G(\lambda) \equiv c_2(\lambda) - F_H(\lambda) = \delta\pi_L Z + (1 - \delta\pi_L)c_2(\lambda) - (1 - \pi_H)(1 - \theta)d(1 - \lambda)X - U_L^* \quad (9)$$

The result says that the high-type bank optimally adopts a resolution policy with a liquidation bias and the bias is strict if only if $F_H^* < c_2(\lambda^{EF})$. When $F_H^* \geq c_2(\lambda^{EF})$, the optimal debt defaults whenever the bad state occurs. In this case, the debt becomes a claim of total cash flow from the loan pool and is thus a linear security. By Lemma 1, the value of the debt security is maximised at $\lambda^{EF}$. In addition, the bank retains no cash flow in the bad state, hence distortion in the resolution policy serves no signaling by Lemma 3. Therefore, the equilibrium

[29] Technically, the cash flow distribution in our model satisfies the Hazard Rate Ordering (HRO) property, which is weaker than the Monotone Likelihood Ratio Property (MLRP) commonly assumed in signalling environments. DeMarzo et al. (2015) show that the (HRO) is a sufficient condition to ensure that debt is the optimal monotone security in a signalling framework with liquidity needs.

resolution policy is the efficient one.

On the other hand, when $F_H^* < c_2(\lambda^{EF})$, the equilibrium resolution policy is distorted towards liquidation. This is for two reasons. First, in this case, the optimal debt only defaults in the re-default state, but not in the recovery state. The security is thus concave in the underlying cash flow of the loan pool in the bad state. By Lemma 1, more liquidation is desirable even without information concerns because it realises more gains from trade, i.e. $\lambda^{SI}(\mathcal{F}_H^*) > \lambda^{EF}$. Second, by Lemma 3, further biasing the resolution policy towards liquidation to $\lambda_H^* \geq \lambda^{SI}(\mathcal{F}_H^*)$ can serve as a signal because it is costly for the high-type bank and costlier for the mimicking low-type bank. The signalling effect is at play, i.e the above inequality is strict, unless it is prevented by the binding limited liability constraint.[30]

To summarise, Proposition 2 states that information asymmetry leads to the retention of a convex claim and a liquidation bias in the resolution policy. The bias is strict when $G(\lambda^{EF}) > 0$, which is the case if i) the adverse selection problem is severe, i.e. $\pi_H$ is high or $\pi_L$ is low, and/or ii) the cost of retention $(1-\delta)$ is low, i.e. when retention is ineffective in signalling information.

Finally, the following comparative statics further illustrates the effect of adverse selection on the high-type bank's resolution policy. An increase in the high-type bank's loan quality, $\pi_H$, exacerbates the information friction because it creates stronger incentives for the low-type bank to mimic. This then leads to a larger liquidation bias in the high-type bank's resolution policy in equilibrium.

**Corollary 1.** *The liquidation bias in the high-type bank's resolution policy is increasing in the quality of its loans. That is,* $\frac{\partial \lambda_H^*}{\partial \pi_H} \geq 0$*, where the inequality is strict if and only if* $G(\lambda^{EF}) > 0$.

## 4 Screening and resolution policy

In this section, we study the implication of the bank's distortion in the resolution policy for the bank's screening incentives. We extend the model to incorporate a loan-origination stage $t = 0$, at which point the bank can exert non-observable costly screening effort to increase the probability $\gamma$ of receiving a high-quality loan pool at $t = 1$. The main finding is that while information asymmetry leads to underinvestment in screening effort, adopting the optimal resolution policy with a liquidation bias mitigates this underinvestment problem and the associated inefficiency.

[30] The case with $\lambda_H^* > \lambda^{SI}(\mathcal{F}_H^*)$ corresponds to the case 3 in the proof of Proposition 2.

At $t = 0$, the bank is endowed with 1 unit of funds and can invest in a loan pool. When investing, the bank can exert non-observable effort to affect $\gamma \in [\underline{\gamma}, \bar{\gamma}]$, the probability that the loan pool is of high quality at $t = 1$, where $0 \leq \underline{\gamma} < \bar{\gamma} \leq 1$. Such effort can be interpreted as, for example, time and resources spent to assess the quality of the borrowers' investment projects and to screen out borrowers who have less valuable, distress-prone projects. Effort incurs a quadratic cost of $\frac{1}{2}k(\gamma - \underline{\gamma})^2$. We assume $k \geq \frac{U_H^{SI} - U_L^{SI}}{\bar{\gamma} - \underline{\gamma}}$ to guarantee an interior optimal level of effort, and $U_L^{SI} \geq 1$ so that investing in the loan pool is always efficient.

## 4.1 Equilibrium screening effort

The bank is willing to exert costly effort because the expected payoff to the high type $U_H$ is higher than that to the low type $U_L$. Because $U_H$ and $U_L$ are potentially affected by the information environment, the security design, and the resolution policy in the subsequent stages of the model, so is the bank's optimal screening effort.

Notice that since the subsequent equilibrium in the funding stage is separating, the equilibrium payoffs $\{U_H, U_L\}$ do not depend on $\gamma$. We can therefore consider any generic pair of $\{U_H, U_L\}$ that represents the expected payoffs to the bank in the separating equilibrium in the subgame starting at $t = 1$. At $t = 0$, the bank chooses the optimal level of effort to maximise its ex ante expected payoff

$$\max_{\gamma} \quad \gamma U_H + (1 - \gamma) U_L - \frac{1}{2}k(\gamma - \underline{\gamma})^2 \tag{10}$$

The optimal effort is thus

$$\gamma^*(U_H, U_L) = \underline{\gamma} + \frac{U_H - U_L}{k} \tag{11}$$

The optimal effort chosen by the bank is increasing in the difference in the expected payoff $(U_H - U_L)$ between having a high-quality and a low-quality pool. We will look at how this difference changes under symmetric and asymmetric information, and under different resolution policies.

Under symmetric information, both banks with high- and low-quality pools adopt the efficient resolution policy $\lambda^{EF}$ and achieve payoff $(U_H^{SI}, U_L^{SI})$ respectively. Under asymmetric information, the low-type bank attains the same payoff as under symmetric information, i.e. $U_L^* = U_L^{SI}$, because it suffers no information friction and hence optimally chooses the efficient resolution policy $\lambda^{EF}$ and sells a full pass-through security. On the other hand, the high type is strictly worse off under asymmetric information because of the signalling cost $U_H^* < U_H^{SI}$. As a result, the bank exerts *strictly less* effort.

**Lemma 5.** *Compared to the symmetric information case, the bank under-expends screening effort under asymmetric information. That is* $\gamma^*(U_H^*, U_L^*) < \gamma^*(U_H^{SI}, U_L^{SI})$.

### 4.2 Inefficiency of intervention in resolution policy

Next we turn to the question of how regulatory interventions in banks' resolution policies can affect banks' screening effort. As shown in our main result, a resolution policy with a liquidation bias can reduce the high-type bank's funding cost. By Eq. 11, a higher payoff to the high-type bank incentivises the bank to screen borrowers in order to create a high-quality loan pool, further restoring welfare. The following proposition summarises the effect of a regulatory intervention in the resolution policy on banks' ex ante screening effort and on welfare.

**Proposition 3.** *If the government imposes the efficient resolution policy* $\lambda^{EF}$*, the bank exerts less screening effort at* $t = 0$*, hence reducing welfare.*

Proposition 3 suggests that there is an unintended consequence of government policy like Home Affordable Modification Program (HAMP) which aims to restore efficiency in the resolution decision of delinquent mortgages. When there is information asymmetry and banks have financing needs, imposing the efficient resolution policy on the bank *reduces* its payoff in the case of receiving a high-quality loan pool due to adverse selection. This in turn weakens its incentive to exert screening effort to obtain a high-quality pool, decreasing welfare.

## 5 Extension: multiple types

The goal of this section is to show that our result is robust to an extension to multiple types. In line with the baseline two-type model, we find that the liquidation bias is (weakly) larger for banks with higher-quality loan pools.

We extend the baseline model with two types to $n$ types. That is, the probability that the loan pool enters the good ($G$) state is given by $\pi_i \in \{\pi_1, \pi_2, ..., \pi_n\}$, where $1 > \pi_i > \pi_{i-1} > 0$ for all $i \in \{2, ..., n\}$. As before, we focus on the least cost separating equilibrium.[31] Analogous

[31] The prior distribution of types is thus irrelevant.

to Eq. 8, the least cost separating equilibrium with $n$ types is given by

$$\begin{aligned} U_1^n &= \max_{(\mathcal{F}_1,\lambda_1)} \quad p_1(\mathcal{F}_1,\lambda_1) + \delta\left[V_1(\lambda_1) - p_1(\mathcal{F}_1,\lambda_1)\right] \\ U_i^n &= \max_{(\mathcal{F}_i,\lambda_i)} \quad p_i(\mathcal{F}_i,\lambda_i) + \delta\left[V_i(\lambda_i) - p_i(\mathcal{F}_i,\lambda_i)\right] \quad s.t. \quad (IC_i) \quad \forall\, i \geq 2 \end{aligned} \tag{12}$$

where the incentive compatibility constraint that type $i-1$ will not mimic type $i$ is given by

$$(IC_i) \quad U_{i-1}^n \geq p_i(\mathcal{F}_i,\lambda_i) + \delta\left[V_{i-1}(\lambda_i) - p_{i-1}(\mathcal{F}_i,\lambda_i)\right] \tag{13}$$

We show in the Appendix that the set of local incentive compatibility constraints $(IC_i)$ implies no mimicking by all other types, and thus characterises the equilibrium. We summarise the result in the following proposition and discuss the intuition below. We denote by superscript $n$ all equilibrium quantities in a model with $n$ types.

**Proposition 4.** *In a model with $n$ types, the least cost separating equilibrium exists, in which the bank of type $i$ issues risky debt with a promised repayment $F_i^n$. There exists a unique type $j > 1$ such that the equilibrium resolution policies are*

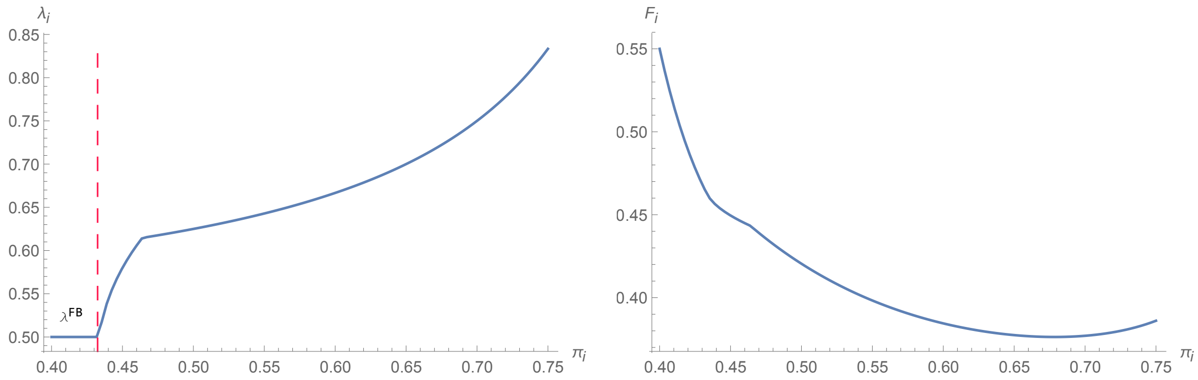
$$\lambda_i^n \begin{cases} = \lambda^{EF} & \textit{for} \quad i < j \\ > \lambda^{EF} & \textit{for} \quad i \geq j \end{cases}$$

*Moreover, $\lambda_i^n \geq \lambda_{i-1}^n$ for all $i \geq j$, where the inequality is strict whenever $\lambda_i^n < 1$.*

Proposition 4 shows that the insights from our baseline two-type model (Proposition 2) can be extended to a model with multiple types. That is, banks with higher-quality loans adopt resolution policies with (weakly) larger liquidation bias. The intuition behind this result is similar to the two-type case. Banks with higher-quality loans $(i \geq j)$ who face severe adverse selection retain larger amounts of the loan pool's cash flow $F_i^n < c_2(\lambda^{EF})$. These banks issue debt securities that are concave in the cash flow of the loan pool in the bad state, as discussed in Section 3.2, and therefore distort their resolution policies in equilibrium towards liquidation. Banks with lower-quality loans $(i < j)$ adopt the efficient resolution policy and separate by retaining more cash flow of the loan pool.

This extension sheds additional light on the interaction between the bank's security design and the choice of resolution policy. Banks with higher-quality loans $(i \geq j)$ signal their quality

Figure 2: The equilibrium resolution policy $\lambda_i^n$ (left panel) and promised repayment $F_i^n$ (right panel). The parameter values used in this plot are $Z = 0.55$, $(1-d)Z = 0.2$, $dX = 0.3$, $\theta = 0.5$, $\delta = 0.9$, $\pi_1 = 0.4$, $\pi_{100} = 0.75$, $\pi_i - \pi_{i-1} = \frac{0.35}{99}$ for all $i \in \{2, ..., 100\}$, and $\mathcal{L}(\lambda) = \frac{1-(1-\lambda)^2}{2}X$. The red dashed line in the left panel marks the type $j$.




through a combination of cash flow retention and distortion in the resolution policy. In contrast to DeMarzo (2005) and DeMarzo et al. (2015) who only allow retention as a signal, we find that cash flow retention by banks in equilibrium may be non-monotonic in loan quality. This is because a liquidation bias in the resolution policy can substitute retention as a signal of quality in our model. This result is formally presented in Corollary 2 and illustrated in Figure 2.

**Corollary 2.** *In equilibrium, $F_i^n$ is strictly decreasing in loan quality $i$ for all $i < j$. $F_i^n$ may be non-monotonic in loan quality $i$ for $i \geq j$.*

# 6 Conclusion

This paper investigates whether and how the adverse selection friction in banks' funding would affect their choices of resolution policy for potential distressed borrowers. We show that in order to reduce their funding costs, it could be optimal for banks to adopt a biased resolution policy. Our analysis further highlights the importance of security design for resolution policy, by showing that the optimal resolution policy is biased towards liquidation (continuation) when the issued security is concave (convex). For a normative perspective, our results caution against policies that aim to restore efficiency in resolution, as they can have the unintended consequence of reducing the banks' ex ante screening effort, thereby worsening the average quality of the loan pools and reducing social welfare.

We conclude with some conjectures for directions for future work and extensions. First, this framework can be extended to a setting with multiple banks to study the spillover effects of distressed loan liquidation due to fire-sale or information externality. It could also be fruitful to analyse, in a general equilibrium, the potential impact of banks' resolution policy on the quantity, quality, and the prices of loans originated. Finally, a dynamic framework could shed light on how banks' incentive to liquidate loans vary across business cycles.

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# Appendices

## A Monotone security under limited liability

We restrict to monotone securities under limited liability. In this section we formalise these constraints as follows

$$\begin{aligned}
(MNO) \quad & f_1 \geq f_2(c_2) \geq f_3(c_3) \geq 0 \;\forall c_2 \in [c_2(1), c_2(0)] \text{ and } c_3 \in [c_3(0), c_3(1)] \text{ and} \\
& \text{and } \frac{\partial f_j(c_j)}{\partial c_j} \geq 0 \;\forall j \in \{2,3\} \\
(MNI) \quad & c_1 - f_1 \geq c_2 - f_2(c_2) \geq c_3 - f_3(c_3) \;\forall c_2 \in [c_2(1), c_2(0)] \text{ and } c_3 \in [c_3(0), c_3(1)] \\
& \text{and } \frac{\partial}{\partial c_j}\left(c_j - f_j(c_j)\right) \geq 0 \;\forall j \in \{2,3\}
\end{aligned} \tag{14}$$

where $(MNO)$ and $(MNI)$ stand for the monotonicity constraints of the outside investors and the insider respectively. These constraints state that, respectively, the payoff of the security and the residual payoff to the bank are weakly increasing in the realisation of the cash flow. Note by restricting the payoff of the security and the residual payoff to the bank to be positive, the limited liability constraint is satisfied.

## B Proofs

### B.1 Proof of Lemma 1

$U_i(\mathcal{F}, \lambda; \pi_i)$ is defined in Eq. 8, where $p_i(\mathcal{F}, \lambda)$ and $V_i(\lambda)$ are given by Eq. 1 and 2 respectively. Therefore

$$\begin{aligned}
\frac{\partial U_i(\mathcal{F}, \lambda; \pi_i)}{\partial \lambda} &= \delta \frac{\partial V_i(\lambda)}{\partial \lambda} + (1-\delta)\frac{\partial p_i(\mathcal{F}, \lambda)}{\partial \lambda} \\
&= (1-\pi_i)\left(\left[\delta + (1-\delta) f_2'(c_2(\lambda))\right]\theta c_2'(\lambda) + \left[\delta + (1-\delta) f_3'(c_3(\lambda))\right](1-\theta) c_3'(\lambda)\right)
\end{aligned} \tag{15}$$

- If in the bad state $\mathcal{F}$ is concave, then

$$\frac{\partial U_i(\mathcal{F}, \lambda; \pi_i)}{\partial \lambda} = (1-\pi_i)\left[\delta + (1-\delta) f_3'(c_3(\lambda))\right]\left(\underbrace{\frac{\delta + (1-\delta) f_2'(c_2(\lambda))}{\delta + (1-\delta) f_3'(c_3(\lambda))}}_{\leq 1}\theta c_2'(\lambda) + (1-\theta) c_3'(\lambda)\right)$$

In this case, $\frac{\partial U_i(\mathcal{F},\lambda;\pi_i)}{\partial\lambda} \geq (1-\pi_i)\left[\delta+(1-\delta)f_3'(c_3(\lambda))\right]\left[\theta c_2'(\lambda)+(1-\theta)c_3'(\lambda)\right] \geq 0$ for all $\lambda \leq \lambda^{EF}$, where the first inequality is strict for $\lambda^{EF}$ if $f_3'(c_3(\lambda^{EF})) > f_2'(c_2(\lambda^{EF}))$ and the second inequality is strict for all $\lambda > \lambda^{EF}$. This implies that $\lambda^{SI}(\mathcal{F}) \geq \lambda^{EF}$, where the inequality is strict if $f_3'(c_3(\lambda^{EF})) > f_2'(c_2(\lambda^{EF}))$.

Assuming that the second order condition holds, $\lambda^{SI}(\mathcal{F})$ and thus $\left|\lambda^{SI}(\mathcal{F})-\lambda^{EF}\right|$ is decreasing in $\delta$, because

$$\frac{\partial^2 U_i(\mathcal{F},\lambda^{SI}(\mathcal{F});\pi_i)}{\partial\lambda\partial\delta} = (1-\pi_i)\left[\delta+(1-\delta)f_3'(c_3(\lambda))\right]\frac{\left[f_3'(c_3(\lambda))-f_2'(c_2(\lambda))\right]\theta c_2'(\lambda)}{\left[\delta+(1-\delta f_3'(c_3(\lambda))\right]^2} \leq 0$$

- If in the bad state $\mathcal{F}$ is convex, then

$$\frac{\partial U_i(\mathcal{F},\lambda;\pi_i)}{\partial\lambda} = (1-\pi_i)\left[\delta+(1-\delta)f_2'(c_2(\lambda))\right]\left(\theta c_2'(\lambda)+\underbrace{\frac{\delta+(1-\delta)f_3'(c_3(\lambda))}{\delta+(1-\delta)f_2'(c_2(\lambda))}}_{\leq 1}(1-\theta)c_3'(\lambda)\right)$$

In this case, $\frac{\partial U_i(\mathcal{F},\lambda;\pi_i)}{\partial\lambda} \leq (1-\pi_i)[\delta+(1-\delta)f_3'(c_3(\lambda))]\left[\theta c_2'(\lambda)+(1-\theta)c_3'(\lambda)\right] \leq 0$ for all $\lambda \geq \lambda^{EF}$, where the first inequality is strict for $\lambda^{EF}$ if $f_3'(c_3(\lambda^{EF})) < f_2'(c_2(\lambda^{EF}))$ and the second inequality is strictly for all $\lambda > \lambda^{EF}$. This implies that $\lambda^{SI}(\mathcal{F}) \leq \lambda^{EF}$, where the inequality is strict if if $f_3'(c_3(\lambda^{EF})) < f_2'(c_2(\lambda^{EF}))$.

Assuming that the second order condition holds, $\lambda^{SI}(\mathcal{F})$ is increasing and thus $\left|\lambda^{SI}(\mathcal{F})-\lambda^{EF}\right|$ is decreasing in $\delta$, because

$$\frac{\partial^2 U_i(\mathcal{F},\lambda^{SI}(\mathcal{F});\pi_i)}{\partial\lambda\partial\delta} = (1-\pi_i)\left[\delta+(1-\delta)f_2'(c_2(\lambda))\right]\frac{\left[f_2'(c_2(\lambda))-f_3'(c_3(\lambda))\right](1-\theta)c_3'(\lambda)}{\left[\delta+(1-\delta f_2'(c_2(\lambda))\right]^2} \geq 0$$

- If in the bad state $\mathcal{F}$ is linear, then

$$\begin{aligned}\frac{\partial U_i(\mathcal{F},\lambda;\pi_i)}{\partial\lambda} &= (1-\pi_i)\left[\delta+(1-\delta)f_3'(c_3(\lambda))\right]\left(\underbrace{\frac{\delta+(1-\delta)f_2'(c_2(\lambda))}{\delta+(1-\delta)f_3'(c_3(\lambda))}}_{=1}\theta c_2'(\lambda)+(1-\theta)c_3'(\lambda)\right)\\ &= (1-\pi_i)\left[\delta+(1-\delta)f_3'(c_3(\lambda))\right]\left[\theta c_2'(\lambda)+(1-\theta)c_3'(\lambda)\right]\end{aligned}$$

In this case, $\frac{\partial U_i(\mathcal{F},\lambda;\pi_i)}{\partial\lambda} > 0$ for all $\lambda < \lambda^{EF}$ and $\frac{\partial U_i(\mathcal{F},\lambda;\pi_i)}{\partial\lambda} < 0$ for all $\lambda > \lambda^{EF}$. This implies that $\lambda^{SI}(\mathcal{F}) = \lambda^{EF}$.

## B.2 Proof of Proposition 1

This result follows immediately from the discussion.

## B.3 Proof of Lemma 3

Recall that, Lemma 1 implies that $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \lambda} < 0$ if and only if $\lambda > \lambda^{SI}(\mathcal{F})$ if in the bad state $\mathcal{F}$ is concave, convex or linear.

The single crossing condition is formally defined as the monitonicity of the ratio $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ in type $i$. Using Eq. 1, $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \hat{\pi}} = f_1 - [\theta f_2(c_2(\lambda)) + (1-\theta) f_3(c_3(\lambda))]$, which is strictly greater than 0 by $(MNO)$ given by Eq. 14, and is independent of $i$. Therefore the ratio $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ is monotonic in $i$ if and only if $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})}{\partial \lambda}$ is monotonic in $i$, which is the case if and only if $\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda}$ is monotonic in $i$, where

$$\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda} = (1-\pi_i)\left[\theta(1-f_2'(c_2(\lambda)))c_2'(\lambda) + (1-\theta)(1-f_3'(c_3(\lambda)))c_3'(\lambda)\right]$$

Notice that $1-f_2'(c_2), 1-f_3'(c_3) \in [0,1]$ by $(MNI)$ given by Eq. 14.

- For all $\mathcal{F} \neq \{f_1, c_2(\lambda), c_3(\lambda)\}$, if in the bad state $\mathcal{F}$ is concave, then

$$\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda} = (1-\pi_i)(1-f_2'(c_2(\lambda)))\left[\theta c_2'(\lambda) + (1-\theta)\underbrace{\frac{1-f_3'(c_3(\lambda))}{1-f_2'(c_2(\lambda))}}_{\leq 1} c_3'(\lambda)\right]$$

  In this case, $\theta c_2'(\lambda) + (1-\theta)\frac{1-f_3'(c_3(\lambda))}{1-f_2'(c_2(\lambda))}c_3'(\lambda) \leq \theta c_2'(\lambda) + (1-\theta)c_3'(\lambda) \leq 0$ for all $\lambda \geq \lambda^{EF}$, where the first inequality is strict for $\lambda^{EF}$ if $f_3'(c_3(\lambda^{EF})) > f_2'(c_2(\lambda^{EF}))$ and the second inequality is strict for all $\lambda > \lambda^{EF}$. Therefore $\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda}$ and thus $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial \hat{\pi}}$ are strictly increasing in $i$ for all $\lambda > \lambda^{EF}$, and for $\lambda = \lambda^{EF}$ if $f_3'(c_3(\lambda^{EF})) > f_2'(c_2(\lambda^{EF}))$. This implies that distortion towards liquidation beyond the symmetric-information benchmark could signal quality.

- For all $\mathcal{F} \neq \{f_1, c_2(\lambda), c_3(\lambda)\}$, if in the bad state $\mathcal{F}$ is convex, then

$$\frac{\partial V_i(\lambda)}{\partial \lambda} - \frac{\partial p_i(\mathcal{F},\lambda)}{\partial \lambda} = (1-\pi_i)(1-f_3'(c_3(\lambda)))\left[\theta \underbrace{\frac{1-f_2'(c_2(\lambda))}{1-f_3'(c_3(\lambda))}}_{\leq 1} c_2'(\lambda) + (1-\theta)c_3'(\lambda)\right]$$

In this case, $\theta\frac{1-f_2'(c_2(\lambda))}{1-f_3'(c_3(\lambda))}c_2'(\lambda)+(1-\theta)c_3'(\lambda)>\theta c_2'(\lambda)+(1-\theta)c_3'(\lambda)\geq 0$ for all $\lambda\leq\lambda^{EF}$, where the first inequality is strict for $\lambda^{EF}$ if $f_3'(c_3(\lambda^{EF}))<f_2'(c_2(\lambda^{EF}))$ and the second inequality is strict for all $\lambda<\lambda^{EF}$. Therefore $\frac{\partial V_i(\lambda)}{\partial\lambda}-\frac{\partial p_i(\mathcal{F},\lambda)}{\partial\lambda}$ and thus $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial\lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial\hat{\pi}}$ are strictly decreasing in $i$ for all $\lambda<\lambda^{EF}$, and for $\lambda=\lambda^{EF}$ if $f_3'(c_3(\lambda^{EF}))<f_2'(c_2(\lambda^{EF}))$. This implies that distortion towards continuation beyond the symmetric-information benchmark could signal quality.

- Tbove analysis implies that, for all $\mathcal{F}\neq\{f_1,c_2(\lambda),c_3(\lambda)\}$, if in the bad state $\mathcal{F}$ is linear, then $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial\lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial\hat{\pi}}$ is strictly increasing in $i$ for $\lambda>\lambda^{EF}$ and strictly decreasing in $i$ for $\lambda<\lambda^{EF}$. This implies that distortion towards both liquidation and continuation beyond the symmetric-information benchmark could signal quality.

- If $\mathcal{F}=\{f_1,c_2(\lambda),c_3(\lambda)\}$, then $\frac{\partial V_i(\lambda)}{\partial\lambda}-\frac{\partial p_i(\mathcal{F},\lambda)}{\partial\lambda}=0$ and $\frac{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial\lambda}{\partial U_i(\mathcal{F},\lambda;\hat{\pi})/\partial\hat{\pi}}$ is constant in $i$. Therefore the single crossing condition is not satisfied, and distortion in the resolution policy does not signal information.

## B.4 Proof of Lemma 2, Lemma 4, and Proposition 2

We establish the proofs of these related results in two steps. We first solve for the least cost separating (LCS) equilibrium which is characterised by the maximisation problem stated in Eq. 8 (Lemma 4 and Proposition 2). Then we show that only the LCS equilibrium survives the Intuitive Criterion (Lemma 2).

In the least cost separating equilibrium is characterised by the opitimisation problem stated in Eq. 8, as discussed in the main text. Here we re-write the problem as follows, with explicit constraint for a monotone security under limited liability.

$$\begin{aligned} & max_{(\mathcal{F}_H,\lambda_H)} \quad U_H(\mathcal{F}_H,\lambda_H;\pi_H) \\ s.t. \quad & (IC) \quad U_L^{SI}\geq U_L(\mathcal{F}_H,\lambda_H;\pi_H) \text{ and} \\ & (MNO) \text{ and } (MNI) \text{ given by Eq. 14} \end{aligned} \tag{16}$$

In what follows, since the resolution policy $\lambda_H$ is pre-committed, only three cash flow occur in equilibrium, namely $c_1$, $c_2(\lambda_H)$ and $c_3(\lambda_H)$.[32] For brevity we suppress the dependency of

[32] The equilibrium security is uniquely defined for these cash flow that occur in equilibrium. Although the payoff of the optimal security may not be uniquely pinned down for the cash flow associated with off-equilibrium resolution policies, this is inconsequential for solving the optimal resolution policy.

$f_j(c_j)$ on $c_j$, $j \in \{2, 3\}$, whenever it is clear.

**Lemma 4: The optimality of risky debt**

We can now begin to prove Lemma 4. The proof is constructed by establishing several claims in succession. For any given $\lambda_H$, an optimal security $\mathcal{F}_H(\lambda_H)$ maximises the high-type bank's expected payoff

$$\delta V_H(\lambda_H) + (1-\delta)p_H(\mathcal{F}_H, \lambda_H)$$

subject to the constrains $(IC)$, $(MNO)$ and $(MNI)$. Since $V_H(\lambda_H)$ is not affected by the security design, the security maximises the sales proceeds $p_H(\mathcal{F}_H, \lambda_H) = \pi_H f_1 + (1-\pi_H)[\theta f_2 + (1-\theta) f_3]$. Since $\lambda_H$ plays no role in this proof, we subsequently denote proceeds from the sale of the security by $p_H(\mathcal{F}_H)$ for the ease of notation.

Given a committed resolution policy $\lambda_H$, there can only be three cash flow realisations $c_1$, $c_2(\lambda_H)$, and $c_3(\lambda_H)$ in equilibrium. Denote by $f_1^*$, $f_2^*$, and $f_3^*$ the payoffs of the optimal security for these equilibrium cash flow realisations respectively. Claim 1–4 below aim to establish the properties that the equilibrium payoffs of the optimal security must satisfy. We finally characterise the properties of the full security and show that a risky debt as described in Lemma 4 is indeed an optimal security.

**Claim 1.** *For an optimal security $\mathcal{F}_H(\lambda_H)$, $f_1^* < c_1$.*

*Proof.* If $f_1^* = c_1$, by (MCI), $f_2^* = c_2(\lambda_H)$ and $f_3^* = c_3(\lambda_H)$. This security (full equity) violates (IC). □

**Claim 2.** *For any optimal security $\mathcal{F}_H(\lambda_H)$, the (IC) must bind.*

*Proof.* Suppose instead the $(IC)$ is slack for some optimal security with payoffs $\{f_1^*, f_2^*, f_3^*\}$. By Claim 1, $f_1^* < c_1$. Unless $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, there exists a security $\hat{\mathcal{F}}$ with payoffs $\{\hat{f}_1, f_2^*, f_3^*\}$ with $\hat{f}_1 > f_1^*$ that satisfies the $(IC)$. As $p_H(\cdot)$ strictly increases with $f_1$, $p_H(\hat{\mathcal{F}}) > p_H(\mathcal{F}_H(\lambda_H))$, contradicting the supposition that the security is optimal.

If $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^*$, one can increase the objective function $p_H(\cdot)$ by increasing both $f_1^*$ and $f_2^*$ by some $\epsilon > 0$ without violating the $(IC)$, unless $f_2^* = c_2(\lambda_H)$ or $c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$. Note that $f_2^* = c_2(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 1.

Suppose now $f_1^* < c_1$ and $c_1 - f_1^* = c_2(\lambda_H) - f_2^* = c_3(\lambda_H) - f_3^*$, similarly one can increase all $f_1^*$, $f_2^*$, $f_3^*$ without violating the $(IC)$ to strictly increase $p_H(\cdot)$, unless $f_3^* = c_3(\lambda_H)$. And

$f_3^* = c_3(\lambda_H)$ implies $f_1^* = c_1$ hence violates Claim 1.

Since we have shown that any security with a slack $(IC)$ can be improved upon, the $(IC)$ must be binding at any optimal security. □

**Claim 3.** *For any optimal security $\mathcal{F}_H(\lambda_H)$, $f_1^* > c_3(\lambda_H)$.*

*Proof.* Suppose instead that $f_1^* \leq c_3(\lambda_H)$. By $(MNO)$, $c_3(\lambda_H) \geq f_1^* \geq f_2^* \geq f_3^*$. This implies that the $(IC)$ is slack because the mimicking payoff

$$\delta V_L(\lambda_H) + p_H(\mathcal{F}_H^*) - \delta p_L(\mathcal{F}_H^*) \leq \delta V_L(\lambda_H) + (1-\delta)c_3(\lambda_H) < V_L(\lambda_H) \leq V_L(\lambda^{EF}) = U_L^*$$

By Claim 2, a slack $(IC)$ contradicts the optimality of $\mathcal{F}_H(\lambda_H)$. □

**Claim 4.** *Any optimal security $\mathcal{F}_H(\lambda_H)$ has either*

1. $f_1^* = f_2^* > f_3^* = c_3(\lambda_H)$ *or*

2. $f_1^* > f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$

*Proof.* Consider a security that pays off $\hat{f}_1$, $\hat{f}_2$, and $\hat{f}_3$ for cash flow $c_1$, $c_2(\lambda_H)$ and $c_3(\lambda_H)$ respectively, such that with the $(IC)$ binds. Using the $(IC)$, write $\hat{f}_1$ as a function of $\hat{f}_2$ and $\hat{f}_3$

$$\hat{f}_1(\hat{f}_2, \hat{f}_3) = \frac{(1-\delta)U_L^* - [(1-\pi_H) - \delta(1-\pi_L)](\theta\hat{f}_2 + (1-\theta)\hat{f}_3)}{\pi_H - \delta\pi_L} \tag{17}$$

Substitute this $\hat{f}_1$ into the objective function. After some algebraic manipulation, the objective function becomes

$$\delta V_H + (1-\delta)\left[\frac{\pi_H}{\pi_H - \delta\pi_L}(1-\delta)U_L^* + \delta\frac{\pi_H - \pi_L}{\pi_H - \delta\pi_L}(\theta\hat{f}_2 + (1-\theta)\hat{f}_3)\right] \tag{18}$$

which is strictly increasing in $\hat{f}_2$ and $\hat{f}_3$. Since $\hat{f}_2$ is bounded above by either $c_2(\lambda_H)$ or $\hat{f}_1$, and $\hat{f}_3$ only by $c_3(\lambda_H)$, any optimal security $\mathcal{F}_H^*$ must have $f_3^* = c_3(\lambda_H)$ and $f_2^* = \min\{f_1^*, c_2(\lambda_H)\}$. Finally, by Claim 3, $f_1^* > c_3(\lambda_H)$ and hence $f_2^* > c_3(\lambda_H)$.

□

Having now analysed the properties of an optimal security's equilibrium payoffs $\{f_1^*, f_2^*, f_3^*\}$, we now consider the security's payoffs associated with the off-equilibrium cash flow realisations, i.e. $f_2(c_2)$ and $f_3(c_3)$ $\forall\lambda \in (0,1)$.

**Claim 5.** *For any optimal security $\mathcal{F}_H(\lambda_H)$, $f_3(c_3) = c_3(\lambda)\ \forall\lambda \le \lambda_H$, and either*

1. $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall\lambda \le \lambda_H$, *or*

2. $f_1^* > f_2(c_2) = c_2(\lambda)$ *and* $f_3(c_3) = c_3(\lambda)\ \forall\lambda \ge \lambda_H$

*Proof.* Notice that these payoffs do not affect either the objective function or the $(IC)$. Therefore they are only restricted by the $(MNO)$ and the $(MNI)$. By Claim 4, $f_3^* = c_3(\lambda_H)$. The $(MNI)$ thus implies that $f_3(c_3) = c_3(\lambda)\ \forall\lambda \le \lambda_H$, because $c_3(\lambda)$ is increasing in $\lambda$.

By Claim 4, there are two cases. In the first case, $f_1^* = f_2^*$. The $(MNO)$ then implies that $f_1^* = f_2^* = f_2(c_2(\lambda))\ \forall\lambda \le \lambda_H$, because $c_2(\lambda)$ is decreasing in $\lambda$. In the second case, $f_2^* = c_2(\lambda_H) > f_3^* = c_3(\lambda_H)$. The $(MNI)$ then implies that $f_2(c_2) = c_2(\lambda)$ and $f_3(c_3) = c_3(\lambda)\ \forall\lambda \ge \lambda_H$. □

Finally we can now verify that a risky debt with face value $F_H(\lambda_H) \in (c_3(\lambda_H), c_1)$, as defined in Lemma 4, indeed is an optimal security as it satisfies Claim 1–5.

### Proposition 2: the LCS equilibrium

We prove Proposition 2 by solving the optimisation programme in Eq. 8, which characterises the LCS equilibrium. Our goal is to highlight the properties of the equilibrium security design and resolution policy. Following Lemma 4, the optimal resolution policy is given by

$$\lambda_H^* = \arg\max_{\lambda_H} \quad u(\lambda_H) \equiv U_H(\mathcal{F}_H(\lambda_H), \lambda_H; \pi_H) \tag{19}$$

and the optimal security is given by $\mathcal{F}_H^* = \mathcal{F}_H(\lambda_H^*)$, which is a debt security with a promised repayment $F_H^* = F_H(\lambda_H^*)$.

First, we show that $\mathcal{F}(\lambda_H)$ is given by the binding $(IC)$. We prove this by contradiction. Suppose there the $(IC)$ is slack given $\lambda_H$ and $\mathcal{F}(\lambda_H)$, which is a debt security with a promised repayment $F_H(\lambda_H)$. Then there exists $F'_H > F_H(\lambda_H)$ such that the $(IC)$ is still satisfied given a debt security with a promised repayment $F'_H$. However, the objective function is strictly greater. This contradicts with the optimality of $\mathcal{F}(\lambda_H)$. Therefore any $\mathcal{F}(\lambda_H)$ must bind the $(IC)$, i.e. it is defined by the binding $(IC)$. For any $\lambda_H$, there can be two cases:

(i) $F_H(\lambda_H) \in [(1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X, Z)$ if and only if $G(\lambda_H) \le 0$, or

(ii) $F_H(\lambda_H) \in ((1-d)Z + d\,\mathcal{L}(\lambda_H), (1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X)$ if and only if $G(\lambda_H) > 0$, where $G(\lambda)$ is given by Eq. 9, derived from Eq. 21.

We then proceed to solve the Eq. 19.

**Case (i):** $G(\lambda_H) \leq 0$ **and** $F_H(\lambda_H) \in [(1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X, Z)$

In this case, the market value of the high type's security is given by

$$p_H(F_H, \lambda_H) = \pi_H F_H + (1-\pi_H)[(1-d)Z + d\,\mathcal{L}(\lambda_H) + d\theta(1-\lambda_H)X] \tag{20}$$

A binding $(IC)$ implies that

$$F_H(\lambda_H) = \frac{U_L^* - \delta\pi_L Z - (1-\pi_H)[(1-d)Z + d\,\mathcal{L}(\lambda_H) + \theta d(1-\lambda_H)X]}{\pi_H - \delta\pi_L} \tag{21}$$

We now show that, the objective function of the resulting univariate optimisation programme, $u(\lambda_H)$, is increasing in $\lambda_H$ if and only if $\lambda_H \leq \lambda^{EF}$. To see this, we differentiate $u(\lambda_H)$ w.r.t. $\lambda_H$:

$$\frac{\partial u(\lambda_H)}{\partial \lambda_H} = (1-\delta)\left[\frac{\partial p_H(F_H(\lambda_H),\lambda_H)}{\partial \lambda_H} + \frac{\partial p_H(F_H(\lambda_H),\lambda_H)}{\partial F_H}\frac{\partial F_H(\lambda_H)}{\partial \lambda_H}\right] + \delta\frac{\partial V_H(\lambda_H)}{\partial \lambda_H} \tag{22}$$

Notice that $\frac{\partial u(\lambda_H^{EF})}{\partial \lambda_H} = 0$ because $\frac{\partial p(F_H,\lambda^{EF})}{\partial \lambda_H} = 0$ and $\frac{\partial F_H(\lambda^{EF})}{\partial \lambda_H} = 0$. Moreover, $u(\lambda_H)$ is strictly concave in $\lambda_H$. After some algebraic manipulation, we have

$$\frac{\partial^2 u(\lambda_H)}{\partial \lambda_H^2} = \frac{\delta(1-\pi_H)(\pi_H - \pi_L)}{\pi_H - \delta\pi_L} d\,\mathcal{L}''(\lambda_H) < 0 \tag{23}$$

Therefore, for all $\lambda_H$ such that $G(\lambda_H) \leq 0$, $u(\lambda_H)$ is increasing in $\lambda_H$ if and only if $\lambda_H \leq \lambda^{EF}$.

**Case (ii):** $G(\lambda_H) > 0$ **and** $F_H(\lambda_H) \in ((1-d)Z + d\,\mathcal{L}(\lambda_H), (1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X)$

In this case, the market value of the high type's security is given by

$$p_H(F_H, \lambda_H) = [\pi_H + (1-\pi_H)\theta]F_H + (1-\pi_H)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_H)] \tag{24}$$

A binding $(IC)$ implies that

$$F_H(\lambda_H) = \frac{\begin{array}{r} U_L^* - \delta\pi_L Z - \delta(1-\pi_L)\theta[(1-d)Z + d\,\mathcal{L}(\lambda_H) + d(1-\lambda_H)X] \\ - (1-\pi_H)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_H)] \end{array}}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} \tag{25}$$

We now show that, there exists $\tilde{\lambda}_H \in (\lambda^{EF}, 1]$, such that $u(\lambda_H)$ is increasing in $\lambda_H$ if and only if $\lambda_H \leq \tilde{\lambda}_H$, which is equivalent to $u(\lambda_H)$ being quasi-concave in $\lambda_H$. To see this, we evaluate the first derivative of $u(\lambda_H)$ w.r.t. $\lambda_H$, given by Eq. 22 using Eq. 24–25:

$$\frac{\partial u(\lambda_H)}{\partial \lambda_H} = (1-\delta)\frac{[\pi_H + (1-\pi_H)\theta]\,[-\delta(1-\pi_L)\theta d(\mathcal{L}'(\lambda_H) - X) - (1-\pi_H)(1-\theta)d\,\mathcal{L}'(\lambda_H)]}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]}$$
$$+ (1-\delta)(1-\pi_H)(1-\theta)d\,\mathcal{L}'(\lambda_H) + \delta(1-\pi_H)d(\mathcal{L}'(\lambda_H) - \theta X)$$

And the second derivative is given by

$$\frac{\partial^2 u(\lambda_H)}{\partial \lambda_H^2} = [(1-\pi_H)(1-\theta) - \theta(1-\delta)] \times \frac{\delta(\pi_H - \pi_L)}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} d\,\mathcal{L}''(\lambda_H) \tag{26}$$

Notice that, depending on the sign of $[(1-\pi_H)(1-\theta) - \theta(1-\delta)]$, $u(\lambda_H)$ can be either concave or convex.

Suppose $[(1-\pi_H)(1-\theta) - \theta(1-\delta)] \leq 0$, then $u(\lambda_H)$ is convex in $\lambda_H$. This implies that $u(\lambda_H)$ is increasing in $\lambda_H$ for all $\lambda_H$, as

$$\frac{\partial u(\lambda_H)}{\partial \lambda_H} > -(1-\delta)\frac{[\pi_H + (1-\pi_H)\theta](1-\pi_H)d(1-\theta)X}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} + (1-\pi_H)(1-\theta)X$$
$$= \delta\frac{[\pi_H + (1-\pi_H)\theta] - [\pi_L + (1-\pi_L)\theta]}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]}(1-\pi_H)d(1-\theta)X > 0$$

where we have used the fact that $\mathcal{L}'(\lambda_H) < X$ for all $\lambda_H$ (Assumption 1) when deriving the first line.

Suppose $[(1-\pi_H)(1-\theta) - \theta(1-\delta)] > 0$, then $u(\lambda_H)$ is concave in $\lambda_H$. At $\lambda_H = \lambda^{EF}$, after some algebraic manipulation using the fact that $\mathcal{L}'(\lambda^{EF}) = \theta X$, we have

$$\frac{\partial u(\lambda^{EF})}{\partial \lambda_H} = \frac{(1-\delta)\delta(\pi_H - \pi_L)\theta d(1-\theta)X}{[\pi_H + (1-\pi_H)\theta] - \delta[\pi_L + (1-\pi_L)\theta]} > 0 \tag{27}$$

Therefore there exists $\tilde{\lambda}_H \in (\lambda^{EF}, 1]$, such that $u(\lambda_H)$ is increasing in $\lambda_H$ if and only if $\lambda \leq \tilde{\lambda}_H$, where $\tilde{\lambda}_H$ is given by $\frac{\partial u(\tilde{\lambda}_H)}{\partial \lambda_H} = 0$ if $\frac{\partial u(1)}{\partial \lambda_H} \leq 0$, and $\tilde{\lambda}_H = 1$ otherwise.

To summarise, there exists $\tilde{\lambda}_H \in (\lambda^{EF}, 1]$, such that for all $\lambda_H$ such that $G(\lambda_H) > 0$, $u(\lambda_H)$ is increasing in $\lambda_H$ if and only if $\lambda_H \leq \tilde{\lambda}_H$. After some algebraic manipulation, $\tilde{\lambda}_H$ can be defined by

$$\tilde{\lambda}_H \begin{cases} \text{is defined by} & \text{if } [(1-\pi_H)(1-\theta)-\theta(1-\delta)]>0 \\ \quad \mathcal{L}'(\tilde{\lambda}_H) = \frac{\delta-[\pi_H+(1-\pi_H)\theta]}{(1-\pi_H)(1-\theta)-\theta(1-\delta)}\theta X, & \quad \text{and } \mathcal{L}'(1) < \frac{\delta-[\pi_H+(1-\pi_H)\theta]}{(1-\pi_H)(1-\theta)-\theta(1-\delta)}\theta X \\ = 1, & \text{otherwise} \end{cases} \tag{28}$$

We can now describe the equilibrium resolution policy $\lambda_H^*$. Notice that $G(\tilde{\lambda}_H) < G(\lambda^{EF})$, where $G(\lambda)$ is given by Eq. 9. This follows because $G(\lambda)$ is decreasing in $\lambda$ for $\lambda \geq \lambda^{EF}$. To see this,

$$\frac{\partial G(\lambda)}{\partial \lambda} = (1-\delta\pi_L)d[\mathcal{L}'(\lambda) - X] + (1-\pi_H)d(1-\theta)X \tag{29}$$

For $\lambda \geq \lambda^{EF}$, $\mathcal{L}'(\lambda) \leq \theta X$, and the above expression is smaller than

$$-(1-\delta\pi_L)d(1-\theta)X + (1-\pi_H)d(1-\theta)X = (\delta\pi_L - \pi_H)d(1-\theta)X < 0 \tag{30}$$

The equilibrium resolution policy is thus characterised as follows

1. If $G(\tilde{\lambda}_H) < G(\lambda^{EF}) \leq 0$, then $\lambda_H^* = \lambda^{SI}(\mathcal{F}_H^*) = \lambda^{EF}$.

   For $\lambda_H \leq \lambda^{EF}$, $u(\lambda_H)$ is increasing in $\lambda_H$ in either case; for $\lambda_H \geq \lambda^{EF}$, $G(\lambda_H) < \lambda^{EF} \leq 0$, and $u(\lambda^{EF})$ is decreasing as described by Case (i). Therefore, $u(\lambda_H)$ is maximised at $\lambda_H^* = \lambda^{EF}$.

   In equilibrium, $F_H^* \geq c_2(\lambda_H^*)$, implying $\lambda^{SI}(\mathcal{F}_H^*) = \lambda^{EF}$. Therefore $\lambda_H^* = \lambda^{SI}(\mathcal{F}_H^*) = \lambda^{EF}$.

2. If $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{EF})$, then $\lambda_H^* = \check{\lambda}_H$, where $\check{\lambda}_H > \lambda^{EF}$ is given by $G(\check{\lambda}_H) = 0$.

   Notice that $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{EF})$ and that $G(\lambda)$ is decreasing in $\lambda$ for $\lambda \geq \lambda^{EF}$ imply that there exists $\check{\lambda}_H \in (\lambda^{EF}, \tilde{\lambda}_H]$ such that $G(\check{\lambda}_H) = 0$. Moreover, $G(\lambda_H) > 0$ for $\lambda_H \in (\lambda^{EF}, \check{\lambda}_H)$ and $G(\lambda_H) < 0$ for $\lambda_H > \check{\lambda}_H$. The result thus follows because, for $\lambda_H \leq \lambda^{EF}$, $u(\lambda_H)$ is increasing in $\lambda_H$ in either case; for $\lambda_H \in (\lambda^{EF}, \check{\lambda}_H)$, $G(\lambda_H) > 0$ and $u(\lambda_H)$ is increasing as described by Case (ii); for $\lambda \geq \check{\lambda}_H > \lambda^{EF}$, $G(\lambda_H) \leq 0$ and $u(\lambda_H)$ is decreasing as described by Case (i). Therefore, $u(\lambda_H)$ is maximised at $\check{\lambda}_H$.

   In equilibrium, $F_H^* = c_2(\lambda_H^*)$, implying that $\lambda^{SI}(\mathcal{F}_H^*) > \lambda^{EF}$ following the proof of Lemma 3. Moreover, $U_i(\mathcal{F}_H^*, \lambda; \pi_i)$ is strictly decreasing in $\lambda$ for all $\lambda > \lambda_H^*$. Therefore $\lambda_H^* \geq \lambda^{SI}(\mathcal{F}_H^*) > \lambda^{EF}$.

3. If $0 < G(\tilde{\lambda}_H) < G(\lambda^{EF})$, then $\lambda_H^* = \tilde{\lambda}_H > \lambda^{EF}$, where $\tilde{\lambda}_H$ is given by Eq. 28.

   For $\lambda \leq \lambda^{EF}$, $u(\lambda_H)$ is increasing in $\lambda_H$ in either case; for $\lambda_H \in (\lambda^{EF}, \tilde{\lambda}_H]$, $G(\lambda_H) > 0$ and $u(\lambda_H)$ is increasing in $\lambda_H$ as described by Case (ii); for $\lambda_H > \tilde{\lambda}_H$, $u(\lambda_H)$ is decreasing in $\lambda_H$ in either case. Therefore, $u(\lambda_H)$ is maximised at $\tilde{\lambda}_H > \lambda^{EF}$.

   In equilibrium, $F_H^* = c_2(\lambda_H^*)$, implying that $\lambda^{SI}(\mathcal{F}_H^*) > \lambda^{EF}$ following the proof of Lemma 3. If $\lambda_H^*$ is characterised by the first order conditions (following from the proof of Proposition 2) given by

$$\begin{aligned}\frac{\partial u(\lambda_H)}{\partial \lambda_H} &= \frac{\partial U_H(F_H(\lambda_H), \lambda_H; \pi_H)}{\partial \lambda} + \frac{\partial U_H(F_H(\lambda_H), \lambda_H; \pi_H)}{\partial F}\frac{\partial F_H(\lambda_H)}{\partial \lambda_H} \\ &= \frac{\partial U_H(F_H(\lambda_H), \lambda_H; \pi_H)}{\partial \lambda} + \frac{\partial U_L(F_H(\lambda_H), \lambda_H; \pi_H)}{\partial \lambda}\frac{\partial U_H(F_H(\lambda_H), \lambda_H; \pi_H)/\partial F}{\partial U_L(F_H(\lambda_H), \lambda_H; \pi_H)/\partial F}\end{aligned}$$

   When evaluated at $(F_H^*, \lambda_H^*)$, we have

$$\frac{\partial U_H(F_H^*, \lambda_H^*; \pi_H)}{\partial \lambda} + \frac{\partial U_L(F_H^*, \lambda_H^*; \pi_H)}{\partial \lambda}\frac{\partial U_H(F_H^*, \lambda_H^*; \pi_H)/\partial F}{\partial U_L(F_H^*, \lambda_H^*; \pi_H)/\partial F} = 0$$

   Notice that $\frac{\partial U_L(F_H^*, \lambda; \pi_H)}{\partial \lambda} < \frac{\partial U_H(F_H^*, \lambda; \pi_H)}{\partial \lambda}$ for $\lambda > \lambda^{FB}$ by Lemma 3. Therefore $\frac{\partial U_H(F_H^*, \lambda_H^*; \pi_H)}{\partial \lambda} < 0$, which implies that $\frac{\partial U_H(F_H^*, \lambda_H^*; \pi_H)}{\partial \lambda} < 0$ for all $\lambda \geq \lambda_H^*$. Therefore $\lambda_H^* > \lambda^{SI}(\mathcal{F}_H^*) > \lambda^{EF}$.

To summarise, the equilibrium resolution policy is $\lambda_H^* \geq \lambda^{SI}(\mathcal{F}_H^*) \geq \lambda^{EF}$, where the second inequality is strict if and only if $G(\lambda^{EF}) > 0$.

**Lemma 2: only the LCS equilibrium survives the Intuitive Criterion**

We prove Lemma 2 in two steps. We will first show no pooling PBE satisfy the Intuitive Criterion. And then we show the same for any separating PBE other than the least cost separating PBE.

The logic of the proof is as follows: for any candidate pooling PBE $(U_H^P, U_L^P)$ with an offer $\{\mathcal{F}^P, \lambda^P\}$, we construct an off-equilibrium pooling offer $\{\mathcal{F}', \lambda^P\}$ that prunes the candidate PBE with Intuitive Criterion. Since we do not involve changing $\lambda^P$ in the following analysis, for the ease of notation we will simply denote an offer with $\mathcal{F}$ whenever it does not create confusion.

We begin by applying the Intuitive Criterion to our two-type model as follows: a PBE fails to satisfy the Intuitive Criterion if there exists an unsent offer $\mathcal{F}'$, such that the type $H$ is strictly better off than at the posited PBE by proposing $\mathcal{F}'$ for all best responses with beliefs focused on $H$, and the type $L$ is strictly better at the posited PBE than at $\mathcal{F}'$ for all best responses for all beliefs in response to $\mathcal{F}'$.

Define $J_H(\mathcal{F}')$ and $J_L(\mathcal{F}')$ as the payoff of the H and L type when they deviate to the off-equilibrium offer $\mathcal{F}'$ under a belief focused on $H$

$$J_H(\mathcal{F}') \equiv p_H(\mathcal{F}') + \delta[V_H - p_H(\mathcal{F}')]$$
$$J_L(\mathcal{F}') \equiv p_H(\mathcal{F}') + \delta[V_L - p_L(\mathcal{F}')] \tag{31}$$

Therefore a pooling PBE $(U_H^P, U_L^P)$ does not satisfy the intuitive criterion if there exists an $\mathcal{F}'$ such that $J_H(\mathcal{F}') > U_H^P$ and $J_L(\mathcal{F}') < U_L^P$.

We begin the proof with establishing some useful properties of any pooling PBE $(U_H^P, U_L^P)$. First, the payoffs can be computed as follows:

$$U_H^P \equiv \bar{p}(\mathcal{F}^P) + \delta[V_H - p_H(\mathcal{F}^P)]$$
$$U_L^P \equiv \bar{p}(\mathcal{F}^P) + \delta[V_L - p_L(\mathcal{F}^P)] \tag{32}$$

where $\bar{p}(\mathcal{F}) = \bar{\pi} f_1 + (1-\bar{\pi})[\theta f_2 + (1-\theta) f_3)]$ and $\bar{\pi} \equiv \gamma \pi_H + (1-\gamma)\pi_L$.

Second, in any pooling PBE that satisfies Intuitive Criterion, both types must attain weakly higher payoffs than the least cost separating (LCS) payoffs $(U_H^*, U_L^*)$. The following claim establishes this property formally.

**Claim 6.** *For any pooling PBE $(U_H^P, U_L^P)$ that satisfies the Intuitive Criterion, $U_H^P \geq U_H^*$ and $U_L^P \geq U_L^*$.*

*Proof.* This claim is proved by contradiction. First of all, $U_L^P < U_L^*$ cannot be a PBE because the low type can always attain at least the LCS payoffs $U_L^*$ by deviating to the efficient offer of the low type.

Suppose now $U_H^P < U_H^*$ and $U_L^P \geq U_L^*$. To invoke the Intuitive Criterion, consider a set of beliefs that all deviations are done by the high type. Then by deviating to $(F_H^*, \lambda_H^*)$, the high type achieves its LCS payoff $U_H^* > U_H^P$ whereas the low type's payoff $p_H(F_H^*, \lambda_H^*) + \delta[V_L(\lambda_H^*) - p_L(F_H^*, \lambda_H^*)]$, is also equal to its LCS payoff $U_L^*$ because $(F_H^*, \lambda_H^*)$ is the solution of the LCS problem in Eq. 8 and the (IC) therein is binding at the solution. Now consider another offer $\{F', \lambda_H^*\}$ with $F' = F_H^* - \epsilon$ for some arbitrarily small and positive $\epsilon$ such that the high type's payoff with this off-equilibrium offer is $U'_H \in (U_H^P, U_H^*)$. Such an $F'$ exists because $U_H^P < U_H^*$ and

$F_H^* > c_3$ (Lemma 4). Finally the low type's payoff with the offer $\{F', \lambda_H^*\}$ is $U_L' < U_L^* \leq U_L^P$. $\square$

The third property is shown in the following claim

**Claim 7.** *In any pooling PBE with offer $\{\mathcal{F}^P, \lambda^P\}$, $f_1^p > c_3(\lambda^P)$.*

*Proof.* Suppose instead $f_1^P \leq c_3(\lambda^P)$. Because of (MNO), $c_3 \geq f_1^P \geq f_2^P \geq f_3^P$

$$\begin{aligned} U_L^P &= \delta V_L(\lambda^P) + (\bar{\pi} - \delta\pi_L) f_1^P + [(1-\bar{\pi}) - \delta(1-\pi_L)][\theta f_2^P + (1-\theta) f_3^P] \\ &\leq \delta V_L(\lambda^P) + (\bar{\pi} - \delta\pi_L)[\theta f_2^P + (1-\theta) f_3^P] + [(1-\bar{\pi}) - \delta(1-\pi_L)][\theta f_2^P + (1-\theta) f_3^P] \\ &\leq \delta V_L(\lambda^P) + (1-\delta) c_3(\lambda^P) < V_L(\lambda^P) \leq V_L(\lambda^{EF}) \equiv U_L^* \end{aligned}$$

which contradicts the fact that $U_L^P \geq U_L^*$. $\square$

We are now equipped to construct the PBE pruning offer $\mathcal{F}'$ for any pooling PBE with offer $\mathcal{F}^P$. First, we parametrise a series of offers with $y$ such that

$$\mathcal{F}(y) = \{f_1^P - y, f_2^P - \max\{y - (f_1^P - f_2^P), 0\}, f_3^P\} \tag{33}$$

for $y \in [0, f_1^P - f_3^P]$. Note that $\mathcal{F}(0) = \mathcal{F}^P$ and the domain of $y$ is non-empty thanks to Claim 7 and $f_3^P \leq c_3(\lambda^P)$ due to limited liability. The rest of the proof involves two claims with the parametrised offer $\mathcal{F}(y)$.

**Claim 8.** *There exists a unique $\tilde{y} \in (0, f_1^P - f_3^P)$ that satisfies $J_L(\mathcal{F}(\tilde{y})) = U_L^P$*

*Proof.* The proof is based on the Intermediate Value Theorem. First, $J_L(\mathcal{F}(\epsilon)) > U_L^P$ with $\epsilon \to 0$ because

$$\begin{aligned} J_L(\mathcal{F}(\epsilon)) - U_L^P &= p_H(\mathcal{F}(\epsilon)) - \bar{p}(\mathcal{F}^P) - \delta[p_L(\mathcal{F}(\epsilon)) - p_L(\mathcal{F}^P)] \\ &= p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0 \quad \text{ as } \epsilon \to 0 \end{aligned}$$

Second, $J_L(\mathcal{F}(f_1^P - f_3^P)) < U_L^P$ as $\mathcal{F}(f_1^P - f_3^P) = \{f_3^P, f_3^P, f_3^P\}$, $f_3^P \leq c_3(\lambda^P)$ due to (LL), and following the same argument as in Claim 7,

$$J_L(\mathcal{F}(f_1^P - f_3^P)) \leq \delta V_L + (1-\delta) c_3(\lambda^P) < V_L(\lambda^P) \leq V_L(\lambda^{EF}) = U_L^* \leq U_L^P$$

Finally, $J_L(\mathcal{F}(y))$ is strictly decreasing and continuous in $y$

$$\frac{\partial J_L(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_L < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_L - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{34}$$

Therefore, the Intermediate Value Theorem applies. □

**Claim 9.** $J_H(\mathcal{F}(\tilde{y})) > U_H^P$

*Proof.* This result relies on two properties:

(i) $J_H(\mathcal{F}(\epsilon)) - U_H^P = J_L(\mathcal{F}(\epsilon)) - U_L^P = p_H(\mathcal{F}^P) - \bar{p}(\mathcal{F}^P) > 0$ as $\epsilon \to 0$;

(ii) $0 > \dfrac{\partial J_H(\mathcal{F}(y))}{\partial y} > \dfrac{\partial J_L(\mathcal{F}(y))}{\partial y}$ for $y \in [0, f_1^P - f_3^P]$

(i) is immediate from the definition of $J_H$ while (ii) from the direct comparison between Eq. 34 and

$$\frac{\partial J_H(\mathcal{F}(y))}{\partial y} = \begin{cases} -\pi_H + \delta\pi_H < 0 & \text{for } y \in [0, f_1^P - f_2^P) \\ (1-\theta)(\delta\pi_H - \pi_H) - \theta(1-\delta) < 0 & \text{for } y \in [f_1^P - f_2^P, f_1^P - f_3^P) \end{cases} \tag{35}$$

These two properties imply that the wedges $J_H - U_H^P$ and $J_L - U_L^P$ are the same when $y$ is arbitrarily close to zero. As $y$ increases, $J_L$ decreases strictly faster then $J_H$ . Therefore, at $\tilde{y}$, the wedge of $J_L - U_L^P$ is zero while the wedge $J_H - U_H^P$ is strictly positive. □

The last step of constructing the PBE pruning $\mathcal{F}'$ is to set $\mathcal{F}' = \mathcal{F}(\tilde{y}+\epsilon_y)$ with an arbitrarily small but positive $\epsilon_y$ such that $J_H(\mathcal{F}') > U_H^P$. This $\epsilon_y$ exists because $J_H(\mathcal{F}(\tilde{y})) > U_H^P$ as in Claim 9. And by the properties of $\tilde{y}$ in Claim 8 and $J_L$, $J_L(\mathcal{F}') < J_L(\mathcal{F}(\tilde{y})) = U_L^P$. As a result, the posited pooling PBE $(U_H^P, U_L^P)$ cannot satisfy the Intuitive Criterion.

The proof for showing that no separating PBE other than the LCS PBE can satisfy Intuitive Criterion is very similar to Claim 6. Consider a separating PBE $(U_H, U_L)$, by definition of LCS, $U_H \leq U_H^*$ and $U_L \leq U_L^*$ with at least one strict inequality. First $U_L$ cannot be strictly less than $U_L^*$ because the low type can always achieve at least $U_L^*$ by adopting the efficient resolution policy. The relevant class of separating PBE is thus with $U_H < U_H^*$ and $U_L = U_L^*$. The remaining argument of the proof follows exactly the same as the one in Claim 6 and therefore is omitted.

### B.5 Proof of Corollary 1

We characterise the comparative statics for the three cases discussed in Appendix B.4.

1. If $G(\tilde{\lambda}_H) < G(\lambda^{EF}) \leq 0$, $\lambda_H^* = \lambda^{EF}$.

2. If $G(\tilde{\lambda}_H) \leq 0 < G(\lambda^{EF})$, $\lambda_H^* = \breve{\lambda}_H$, where $\breve{\lambda}_H$ is given by $G(\breve{\lambda}_H) = 0$. In this case, $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$, because $\frac{\partial G(\lambda_H^*)}{\partial \lambda_H} < 0$ and

$$\frac{\partial G(\cdot)}{\partial \pi_H} = (1-\theta)(1-\lambda)X > 0 \tag{36}$$

3. If $0 < G(\tilde{\lambda}_H) < G(\lambda^{EF})$, $\lambda_H^* = \tilde{\lambda}_H$, where $\tilde{\lambda}_H$ is given by Eq. 28. In this case, $\frac{\partial \lambda_H^*}{\partial \pi_H} > 0$, because the LHS of Eq. 28 is decreasing in $\lambda_H$, and the RHS of Eq. 28 is decreasing in $\pi_H$. The derivative of the RHS of Eq. 28 w.r.t $\pi_H$ is equal to

$$-\frac{1-[\delta+(1-\delta)\theta]}{[(1-\pi_H)(1-\theta)-(1-\delta)\theta]^2} < 0 \tag{37}$$

To summarise, $\frac{\partial \lambda_H^*}{\partial \pi_H} \geq 0$, where the inequality is strict if and only if $G(\lambda^{EF}) > 0$.

### B.6 Proof of Lemma 5

This proposition follows immediately from Eq. 11 and the preceding discussion.

### B.7 Proof of Proposition 3

Denote with $U_i(\lambda)$ the expected payoff obtained by the high-type bank in the least cost separating equilibrium, for a given resolution policy. In this equilibrium, the high-type bank chooses a security to offer at $t = 1$ to maximise its expected payoff, while preventing mimicking from the low type. Formally, $U_i(\lambda)$ is equal to the value of the optimisation programme Eq. 8, given $\lambda_H = \lambda$.

By definition of $\lambda_H^*$ as the optimiser of Eq. 8 , $U_H(\lambda_H^*) = U_H^* > U_H(\lambda_H)$ for any $\lambda_H \neq \lambda_H^*$. Thus the screening effort $\gamma^*$ decreases as

$$\gamma^*(U_H(\lambda_H), U_L^{SI}) < \gamma^*(U_H(\lambda_H^*), U_L^{SI}) \quad \forall \lambda_H \neq \lambda_H^*$$

For efficiency, we only need to look at the bank's expected payoff as the investors are always indifferent. The expected payoff is lower when $\lambda_H^*$ is replaced with $\lambda_H$, i.e.

$$\begin{aligned}
&\gamma^*(U_H(\lambda_H), U_L^{SI})U_H(\lambda_H) + [1-\gamma^*(U_H(\lambda_H), U_L^{SI})]U_L^{SI} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{SI}) \\
&< \gamma^*(U_H(\lambda_H), U_L^{SI})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H), U_L^{SI})]U_L^{SI} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H), U_L^{SI}) \\
&\leq \gamma^*(U_H(\lambda_H^*), U_L^{SI})U_H(\lambda_H^*) + [1-\gamma^*(U_H(\lambda_H^*), U_L^{SI})]U_L^{SI} - \frac{1}{2}k\gamma^{*2}(U_H(\lambda_H^*), U_L^{SI})
\end{aligned}$$

The first inequality comes from $U_H(\lambda_H) < U_H(\lambda_H^*)$ and the second weak inequality follows from the definition of optimal $\gamma^*$. Finally, $\lambda_H^{EF}$ is one of the possible $\lambda_H \neq \lambda_H^*$ if and only if $G(\lambda^{EF}) > 0$, where $G(\lambda)$ is given by Eq. 9.

## B.8 Proof of Proposition 4

We first characterise the solution to Eq. 12 and 13. We then show that this solution subject to a set of local incentive compatibility constraints (Eq. 13) indeed is the equilibrium by showing that it satisfies global incentive compatibility (Claim 10).

It is immediate that for the lowest type $i = 1$, $\mathcal{F}_1^n$ is a pass-through security (or debt with face value $F_1 = Z$), and $\lambda_1^n = \lambda^{EF}$. It then follows from the proof of Lemma 4 that the optimal security for all types $i \geq 2$ is a risky debt with face value $F_i^n \in ((1-d)Z + d\,\mathcal{L}(\lambda_i), Z)$.

We next characterise the equilibrium security $F_i^n$ and resolution policy $\lambda_i^n$ for $i \geq 2$. Following the proof of Proposition 2, the $(IC_i)$ binds in equilibrium for all $i \geq 2$. We can then substitute the binding $(IC_i)$ into the objective function of type $i$ to eliminate $F_i^n$, and solve the resulting univariate optimisation problem. Let $\hat{F}_i(\lambda_i, U_{i-1}^n)$ denote the $F_i$ implied by a binding $(IC_i)$, given that the equilibrium expected payoff to type $i-1$ is equal to $U_{i-1}^n$. Let $u_i(\lambda_i, U_{i-1}^n)$ denote the objective function of the resulting univariate optimisation problem. The solution to the problem characterised by Eq. 8 is equal to $\lambda_i^n = \arg\max_{\lambda_i} u_i(\lambda_i)$ for all $i \geq 2$, where analogous to Eq. 19,

$$u_i(\lambda_i, U_{i-1}^n) = (1-\delta)p_i(\hat{F}_i(\lambda_i, U_{i-1}^n), \lambda_i) + \delta V_i(\lambda_i) \tag{38}$$

There can be two cases for $i \geq 2$:

(i) $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_2(\lambda_i), Z)$ if and only if $G_i(\lambda_i, U_{i-1}^n) \leq 0$, or

(ii) $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_3(\lambda_i), c_2(\lambda_i))$ if and only if $G_i(\lambda_i, U_{i-1}^n) > 0$, where $G_i(\lambda, U)$ is given by

$$G_i(\lambda, U) = c_2(\lambda) - \frac{U - \delta\pi_{i-1}Z - (1-\pi_i)\left[(1-d)Z + d\,\mathcal{L}(\lambda) + d\theta(1-\lambda)X\right]}{\pi_i - \delta\pi_{i-1}} \tag{39}$$

and $c_2(\lambda)$ and $c_3(\lambda)$ are defined in Table 1

**Case (i): $G_i(\lambda_i, U_{i-1}^n) \leq 0$ and $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_2(\lambda_i), Z)$**

In this case, the market value of the type $i$'s security is given by

$$p_i(F_i, \lambda_i) = \pi_i F_i + (1-\pi_i)\left[(1-d)Z + d\,\mathcal{L}(\lambda_i) + d\theta(1-\lambda_i)X\right] \tag{40}$$

A binding $(IC_i)$ implies that

$$\hat{F}_i(\lambda_i, U_{i-1}^n) = \frac{U_{i-1}^n - \delta\pi_{i-1}Z - (1-\pi_i)\left[(1-d)Z + d\,\mathcal{L}(\lambda_i) + d\theta(1-\lambda_i)X\right]}{\pi_i - \delta\pi_{i-1}} \tag{41}$$

After some algebraic manipulation similar to those in the proof of Proposition 2, we have $\frac{\partial u(\lambda^{EF}, U_{i-1}^n)}{\partial \lambda_i} = 0$ and $\frac{\partial^2 u(\lambda_i, U_{i-1}^n)}{\partial \lambda_i^2} < 0$. Therefore, for all $\lambda_i$ and $U_{i-1}^n$ such that $G_i(\lambda_i, U_{i-1}^n) \leq 0$, $u_i(\lambda_i, U_{i-1}^n)$ is increasing in $\lambda_i$ if and only if $\lambda_i \leq \lambda^{EF}$.

**Case (ii): $\hat{F}_i(\lambda_i, U_{i-1}^n) \in [c_3(\lambda_i), c_2(\lambda_i))$**

In this case, the market value of the type $i$'s security is given by

$$p_i(F_i, \lambda_i) = [\pi_i + (1-\pi_i)\theta]F_i + (1-\pi_i)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_i)] \tag{42}$$

A binding $(IC)$ implies that

$$\hat{F}_i(\lambda_i, U_{i-1}^n) = \frac{\begin{array}{r} U_{i-1}^n - \delta\pi_{i-1}Z - \delta(1-\pi_{i-1})\theta[(1-d)Z + d\,\mathcal{L}(\lambda_i) + d(1-\lambda_i)X] \\ - (1-\pi_i)(1-\theta)[(1-d)Z + d\,\mathcal{L}(\lambda_i)] \end{array}}{[\pi_i + (1-\pi_i)\theta] - \delta[\pi_{i-1} + (1-\pi_{i-1})\theta]} \tag{43}$$

After some derivation similar to those in the proof of Proposition 2, we can show that there exists $\tilde{\lambda}_i \in (\lambda^{EF}, 1]$, such that for all $\lambda_i$ and $U_{i-1}^n$ such that $G_i(\lambda_i, U_{i-1}^n) > 0$, $u_i(\lambda_i, U_{i-1}^n)$ is increasing in $\lambda_i$ if and only if $\lambda_i \leq \tilde{\lambda}_i$, where $\tilde{\lambda}_i$ is given by

$$\tilde{\lambda}_i \begin{cases} \text{is defined by} & \text{if } [(1-\pi_i)(1-\theta)-\theta(1-\delta)]>0 \\ \quad \mathcal{L}'(\tilde{\lambda}_i) = \frac{\delta-[\pi_i+(1-\pi_i)\theta]}{(1-\pi_i)(1-\theta)-\theta(1-\delta)}\theta X, & \quad \text{and } \mathcal{L}'(1) < \frac{\delta-[\pi_i+(1-\pi_i)\theta]}{(1-\pi_i)(1-\theta)-\theta(1-\delta)}\theta X \\ = 1, & \text{otherwise} \end{cases} \tag{44}$$

Notice that $G_i(\lambda_i, U_{i-1}^n)$ (Eq. 39) is decreasing in $\lambda_i$ for all $\lambda_i$. Following similar reasoning as those in the proof of Proposition 2, the equilibrium resolution policy for type $i \geq 2$ thus satisfies the following conditions:

1. If $G_i(\tilde{\lambda}_i, U_{i-1}^n) < G_i(\lambda^{EF}, U_{i-1}^n) \leq 0$, then $\lambda_i^n = \lambda^{EF}$.
2. If $G_i(\tilde{\lambda}_i, U_{i-1}^n) \leq 0 < G_i(\lambda^{EF}, U_{i-1}^n)$, then $\lambda_i^n = \check{\lambda}_i(U_{i-1}^n)$, where $\check{\lambda}_i(U_{i-1}^n) > \lambda^{EF}$ is given by $G_i(\check{\lambda}_i(U_{i-1}^n), U_{i-1}^n) = 0$.
3. If $0 < G_i(\tilde{\lambda}_i, U_{i-1}^n) < G_i(\lambda^{EF}, U_{i-1}^n)$, then $\lambda_i^n = \tilde{\lambda}_i$, where $\tilde{\lambda}_i > \lambda^{EF}$ is given by Eq. 44.

Notice that Points 2 and 3 imply that, if $G_i(\lambda^{EF}, U_{i-1}^n) > 0$, then $\lambda_i^n = \max\{\check{\lambda}_i(U_{i-1}^n), \tilde{\lambda}_i\}$.

We now prove the first part of the proposition, that there exists a unique type $j > 1$, such that $\lambda_i^n = \lambda^{EF}$ for all $i \leq j$ and $\lambda_i^n > \lambda^{EF}$ for all $i > j$. Recall that $\lambda_1^n = \lambda^{EF}$ and $F_1^n = Z$. If $G_2(\lambda^{EF}, U_1^n) > 0$, then $j = 1$.

For $G_2(\lambda^{EF}, U_1^n) \leq 0$, notice that for any type $i-1 \geq 2$ such that $G_{i-1}(\lambda^{EF}, U_{i-2}^n) \leq 0$, $\lambda_{i-1} = \lambda^{EF}$, and $U_{i-1}^n = \delta\pi_{i-1}Z + (1-\delta)\pi_{i-1}F_{i-1}^n + (1-\pi_{i-1})[(1-d)Z + d\,\mathcal{L}(\lambda^{EF}) + \theta d(1-\lambda^{EF})X]$. This implies that

$$\begin{aligned} G_i(\lambda^{EF}, U_{i-1}^n) &= (1-d)Z + d\,\mathcal{L}(\lambda^{EF}) + d(1-\lambda^{EF})X \\ &\qquad - \frac{(1-\delta)\pi_{i-1}F_{i-1}^n + (\pi_i - \pi_{i-1})\left[(1-d)Z + \mathcal{L}(\lambda^{EF}) + d\theta(1-\lambda^{EF})X\right]}{\pi_i - \delta\pi_{i-1}} \\ &> (1-d)Z + d\,\mathcal{L}(\lambda^{EF}) + d(1-\lambda^{EF})X - F_{i-1}^n = G_{i-1}(\lambda^{EF}, U_{i-2}^n) \end{aligned} \tag{45}$$

where the last equality follows because $G_{i-1}(\lambda^{EF}, U_{i-2}^n) \leq 0$ implies that $F_{i-1}^n = \hat{F}_{i-1}(\lambda^{EF}, U_{i-2}^n)$ as given by Eq. 41 (Case i). The inequality follows because $F_{i-1}^n > (1-d)Z + d\,\mathcal{L}(\lambda^{EF}) + d\theta(1-\lambda^{EF})X$. This implies that there exists $j$, such that for all types $i \leq j$, $G_i(\lambda^{EF}, U_{i-1}^n) \leq 0$ and $\lambda_i^n = \lambda^{EF}$, and $G_{j+1}(\lambda^{EF}, U_j^n) > 0$.

Having shown that $G_i(\lambda^{EF}, U_{i-1}^n) \leq 0$ for all $i \leq j$, we now show by contradiction that $G_i(\lambda^{EF}, U_{i-1}^n) > 0$ for all $i > j$. Notice that Eq. 45 implies that $G_{j+1}(\lambda^{EF}, U_j^n) > G_j(\lambda^{EF}, U_{j-1}^n) =$

0. Suppose that there exists $k > j+1$ such that $G_k(\lambda^{EF}, U_{k-1}^n) \leq 0$. This implies that $\lambda_{j+1}^n > \lambda^{EF} = \lambda_k^n$, and $F_{j+1}^n < c_2(\lambda_{j+1}^n) < c_2(\lambda_k^n) \leq F_k^n$, where $c_2(\lambda)$ is defined in Table 1. This violates incentive compatibility as the type $j+1$ can profitably deviate to $(F_k^n, \lambda_k^n)$:

$$\begin{aligned}
U_{j+1}^n &= p_{j+1}(F_{j+1}^n, \lambda_{j+1}^n) + \delta\left[V_{j+1}(\lambda_{j+1}^n) - p_{j+1}(F_{j+1}^n, \lambda_{j+1}^n)\right] \\
&= (1-\delta)[\pi_{j+1} + (1-\pi_{j+1})\theta]F_{j+1}^n + \delta\pi_{j+1}Z + \delta(1-\pi_{j+1})\theta c_2(\lambda_{j+1}^n) \\
&\qquad\qquad\qquad\qquad\qquad + (1-\pi_{j+1})(1-\theta)c_3(\lambda_{j+1}^n) \\
&< (1-\delta)\pi_{j+1}F_k^n + \delta\pi_{j+1}Z + (1-\pi_{j+1})\left[\theta c_2(\lambda_{j+1}^n) + (1-\theta)c_3(\lambda_{j+1}^n)\right] \\
&< (1-\delta)\pi_{j+1}F_k^n + \delta\pi_{j+1}Z + (1-\pi_{j+1})\left[\theta c_2(\lambda_k^n) + (1-\theta)c_3(\lambda_k^n)\right] \\
&< p_k(F_k^n, \lambda_k^n) + \delta\left[V_{j+1}(\lambda_k^n) - p_{j+1}(F_k^n, \lambda_k^n)\right]
\end{aligned}$$

Finally, we show that $\lambda_i^n$ is increasing in $i$ for all $i > j$. Recall that, for $i > j$, $\lambda_i^n = \max\{\breve{\lambda}_i(U_{i-1}^n), \tilde{\lambda}_i\}$. If $\lambda_{i-1}^n = \tilde{\lambda}_{i-1}$, then it is immediate that $\lambda_i^n \geq \lambda_{i-1}^n$, since $\tilde{\lambda}_i$ (Eq. 44) is increasing in $\pi_i$, where the inequality is strict whenever $\tilde{\lambda}_{i-1} < 1$. If $\lambda_{i-1}^n = \breve{\lambda}_{i-1}(U_{i-2}^n)$, then $U_{i-1}^n = \delta\pi_{i-1}Z + (1-\delta)\pi_{i-1}F_{i-1}^n + (1-\pi_{i-1})[(1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d\theta(1-\breve{\lambda}_{i-1}(\cdot))X]$. This implies that

$$\begin{aligned}
G_i(\breve{\lambda}_{i-1}(\cdot), U_{i-1}^n) &= (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d(1-\breve{\lambda}_{i-1}(\cdot))X \\
&\qquad - \frac{(1-\delta)\pi_{i-1}F_{i-1}^n + (\pi_i - \pi_{i-1})[(1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d\theta(1-\breve{\lambda}_{i-1}(\cdot))X]}{\pi_i - \delta\pi_{i-1}} \\
&> (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d(1-\breve{\lambda}_{i-1}(\cdot))X - F_{i-1}^n = G_{i-1}(\breve{\lambda}_{i-1}(\cdot), U_{i-2}^n) = 0
\end{aligned}$$

where the inequality follows because $F_{i-1}^n = (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d(1-\breve{\lambda}_{i-1}(\cdot))X > (1-d)Z + d\,\mathcal{L}(\breve{\lambda}_{i-1}(\cdot)) + d\theta(1-\breve{\lambda}_{i-1}(\cdot))X$. This then implies that $\lambda_i^n \geq \breve{\lambda}_i(U_{i-1}^n) > \breve{\lambda}_{i-1}(U_{i-2}^n) = \lambda_{i-1}^n$. To summarise, for all $i > j$, $\lambda_i^n \geq \lambda_{i-1}^n$, where the inequality is strict whenever $\lambda_{i-1}^n < 1$.

Having now characterize the solution to Eq. 12 and 13, we now show that this solution indeed characterises the least cost separating equilibrium.

**Claim 10.** *The optimisation programme subject to global incentive compatibility, defined as*

$$U_i^n \geq p_k(F_k^n, \lambda_k^n) + \delta\left[V_i(\lambda_k^n) - p_i(F_k^n, \lambda_k^n)\right] \quad \forall i, k \in \{1, 2, ..., n\}$$

*is equivalent to the optimisation programme subject to local incentive compatibility ($IC_i$) defined* *by Eq. 13.*

*Proof.* It is immediate that global incentive compatibility implies local incentive compatibility.

It remains to show that the solution to Eq. 12 and 13 indeed satisfies the global incentive compatability constraint. Following the proof of Proposition 2, the $(IC_i)$ binds in equilibrium for all $i \geq 2$. Consider any $k \geq i$. The binding local $(IC_i)$ implies

$$\begin{aligned}
& p_i(F_i^n, \lambda_i^n) \\
\text{by } (IC_i) \ &= p_{i+1}(F_{i+1}^n, \lambda_{i+1}^n) + \delta \left( \left[V_i(\lambda_{i+1}^n) - p_i(F_{i+1}^n, \lambda_{i+1}^n)\right] - \left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right] \right) \\
\text{by } (IC_{i+1}) \ &= p_{i+2}(F_{i+2}^n, \lambda_{i+2}^n) + \delta \left( \left[V_{i+1}(\lambda_{i+2}^n) - p_{i+1}(F_{i+2}^n, \lambda_{i+2}^n)\right] - \left[V_{i+1}(\lambda_{i+1}^n) - p_{i+1}(F_{i+1}^n, \lambda_{i+1}^n)\right] \right) \\
& \quad + \delta \left( \left[V_i(\lambda_{i+1}^n) - p_i(F_{i+1}^n, \lambda_{i+1}^n)\right] - \left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right] \right) \\
\text{by } (IC_s) \ &= p_k(F_k^n, \lambda_k^n) + \delta \sum_{s=i}^{k-1} \left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right] \qquad (46)
\end{aligned}$$

This implies $U_i^n = p_i(F_i^n, \lambda_i^n) + \delta \left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right] \geq p_k(F_k^n, \lambda_k^n) + \delta \left[V_i(\lambda_k^n) - p_i(F_k^n, \lambda_k^n)\right]$ for all $k \geq i$ if

$$\begin{aligned}
& \sum_{s=i}^{k-1} \left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right] \\
& \qquad \geq \left[V_i(\lambda_k^n) - p_i(F_k^n, \lambda_k^n)\right] - \left[V_i(\lambda_i^n) - p_i(F_i^n, \lambda_i^n)\right] \\
& \qquad = \sum_{s=i}^{k-1} \left[V_i(\lambda_{s+1}^n) - p_i(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_i(\lambda_s^n) - p_i(F_s^n, \lambda_s^n)\right]
\end{aligned}$$

which is implied by

$$\begin{aligned}
& \left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right] \\
& \qquad \geq \left[V_i(\lambda_{s+1}^n) - p_i(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_i(\lambda_s^n) - p_i(F_s^n, \lambda_s^n)\right] \quad \forall s \geq i \\
\Leftrightarrow \quad & - F_{s+1}^n + F_s^n - \left[c_2(\lambda_{s+1}^n) - F_{s+1}^n\right]^+ + \left[c_2(\lambda_s^n) - F_s^n\right]^+ \geq 0 \qquad (47)
\end{aligned}$$

Similarly, Eq. 46 implies $U_k^n = p_k(F_k^n, \lambda_k^n) + \delta \left[V_k(\lambda_k^n) - p_k(F_k^n, \lambda_k^n)\right] \geq p_i(F_i^n, \lambda_i^n) + \delta \left[V_k(\lambda_i^n) - p_k(F_i^n, \lambda_i^n)\right]$

for all $k \geq i$ if

$$\sum_{s=i}^{k-1} \left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right]$$
$$\leq \left[V_k(\lambda_k^n) - p_k(F_k^n, \lambda_k^n)\right] - \left[V_k(\lambda_i^n) - p_k(F_i^n, \lambda_i^n)\right]$$
$$= \sum_{s=i}^{k-1} \left[V_k(\lambda_{s+1}^n) - p_k(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_k(\lambda_s^n) - p_k(F_s^n, \lambda_s^n)\right]$$

which is implied by

$$\left[V_s(\lambda_{s+1}^n) - p_s(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_s(\lambda_s^n) - p_s(F_s^n, \lambda_s^n)\right]$$
$$\leq \left[V_k(\lambda_{s+1}^n) - p_k(F_{s+1}^n, \lambda_{s+1}^n)\right] - \left[V_k(\lambda_s^n) - p_k(F_s^n, \lambda_s^n)\right] \quad \forall k \geq s$$
$$\Leftrightarrow \quad -F_{s+1}^n + F_s^n - \left[c_2(\lambda_{s+1}^n) - F_{s+1}^n\right]^+ + \left[c_2(\lambda_s^n) - F_s^n\right]^+ \geq 0 \qquad \Leftrightarrow \text{Eq. } 47$$

Therefore it suffices to show that Eq. 47 is true for all $s$. Following the proof of Proposition 4, there can be three cases.

(i) $s + 1 \leq j$. In this case, $\lambda_{s+1}^n = \lambda_s^n$, and Eq. 47 $\Leftrightarrow F_{s+1}^n < F_s^n$.

(ii) $s \leq j < s + 1$. In this case, $\lambda_{s+1}^n > \lambda_s^n$, and Eq. 47 $\Leftrightarrow -c_2(\lambda_{s+1}^n) + F_s^n \geq 0$, which is true as $-c_2(\lambda_{s+1}^n) + F_s^n > -c_2(\lambda_s^n) + F_s^n \geq 0$.

(iii) $s = 1 > j$. In this case, $\lambda_{s+1}^n > \lambda_s^n$, and Eq. 47 $\Leftrightarrow -c_2(\lambda_{s+1}^n) + c_2(\lambda_s^n) \geq 0$.

□